Course Name	National Diploma: Plant/Animal Production NQF 5 (247 Credits) SAQA ID: 49010/49011
Module Name	Module 4: Farm Finance Learner Guide
Module Code	12400/18400
Unit Standards	116428, 7468, 7483, 7466, 7470, 12417
NQF Level	5
Credits	247

Learner Guide

Animal/Plant Production

Farm Finance

Table of Contents

Dear Learner

This Learner Guide contains all the information to acquire all the knowledge and skills leading to the unit standard:

Unit standard ID:	**Unit standard title:**
116428	Analyse and interpret the financial statements and physical records in an agri-business to generate managerial information
7468	Use mathematics to investigate and monitor the financial aspects of personal, business, national and international issues
7483	Solve problems involving sequences and series in real and simulated situations
7466	Represent and operate on complex numbers in non-trivial situations
7470	Work with a wide range of patterns and inverses of functions and solve related problems
12417	Measure, estimate & calculate physical quantities & explore, critique & prove geometrical relationships in 2 and 3-dimensional space in the life and workplace of adult with increasing responsibilities

You will be assessed during the course of your study. This is called formative assessment. You will also be assessed on completion of this unit standard. This is called summative assessment. Before your assessment, your assessor will discuss the unit standard with you.

It is your responsibility to complete all the exercises in the Assessor Guide. The facilitator will explain the requirements of each exercise with you. You will also be expected to sign a learner contract in your assessor guide. This contract explains responsibility and accountability by both parties.

On the document "Alignment to NQF", you will find information on which qualification this unit standard is linked to if you would like to build towards more credits against this qualification.

Please contact our offices if you would like information with regards to career advising and mentoring services.

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Enjoy the learning experience!

Key to Icons

	Important Information
	Quotes
	Personal Reflection
	Individual Formative Exercise
	Summative Exercise
	Note-pad: Supplementary Information

Alignment to NQF

Element of Programme	
Name of programme	Farm Finance
2. Purpose of the programme	The managers to be able to demonstrate understanding of key concepts of farm finance, to interpret financial statement, to draft forecasts and budgets and to supervise the financial management of a unit.
3. Duration of the programme	10 days, notional hours 210
4. NQF level	5
5. NQF credits	21
6. Specific outcomes	See Unit Standard guide
7. Assessment criteria	See Unit Standard guide
8. Critical cross-field outcomes	See Unit Standard guide
9. Learning assumed to be in place	• Communication at NQF Level 3. • Mathematics at NQF Level 3.

10. Essential embedded knowledge	See Unit Standard guide
11. Range statement	See Unit Standard guide
12. Recognition of Prior Learning (RPL)	Option is available.
13. Learning Materials	Learner Guide, Assessor Guide, Handouts, Facilitator guide, Learner PoE Workbook.
14. Links of the programme to registered unit standards, skills programmes, or qualifications	Registered qualification: Title: National Diploma in Plant/Animal Production ID: 49010/49011 NQF: Level 5 Credits: 247

PART A

Learning Units 1 to 7

Financial Accounting

Learning Unit 1

Understanding the Basics of Accounting

Unit Standard	
116428	**Analyse and interpret the financial statements and physical records in an agri-business to generate managerial information**
Specific Outcomes	
SO1: Use financial and physical information to compile financial statements and physical data to compile physical records	
Learning Outcomes	
• Explain the accounting cycle by means of a diagram. • Explain accounting conventions applied in the financial management of a unit with examples. • Explain the usefulness of physical records in your business	
Organise Collecting Science	Demonstrating Contributing Identifying

WHAT IS THE DIFFERENCE BETWEEN FINANCIAL MANAGEMENT AND ACCOUNTING?

Accounting is about recording how money is spent and financial management is about controlling how money is spent. Financial management requires an understanding of accounting records in order to exert better control.

In this learning unit, you will learn some basics of accounting records and then you will learn how to analyse these records and make decisions accordingly.

THE BASICS OF ACCOUNTING

A lot of people find 'numbers' daunting simply because they do not have the accounting background in order to understand how the numbers got there in the first place.

This section will take you through the basics of how accountants actually create the financial reports, which include things like a Balance Sheet, Income Statement and Cash Flow Statement.

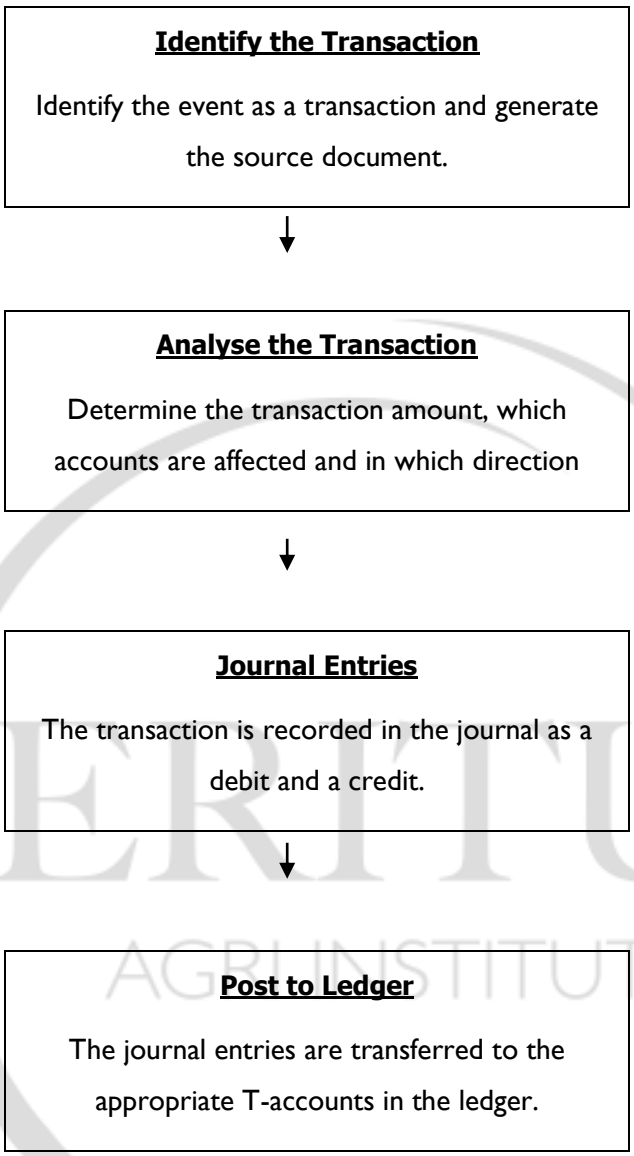
THE ACCOUNTING CYCLE

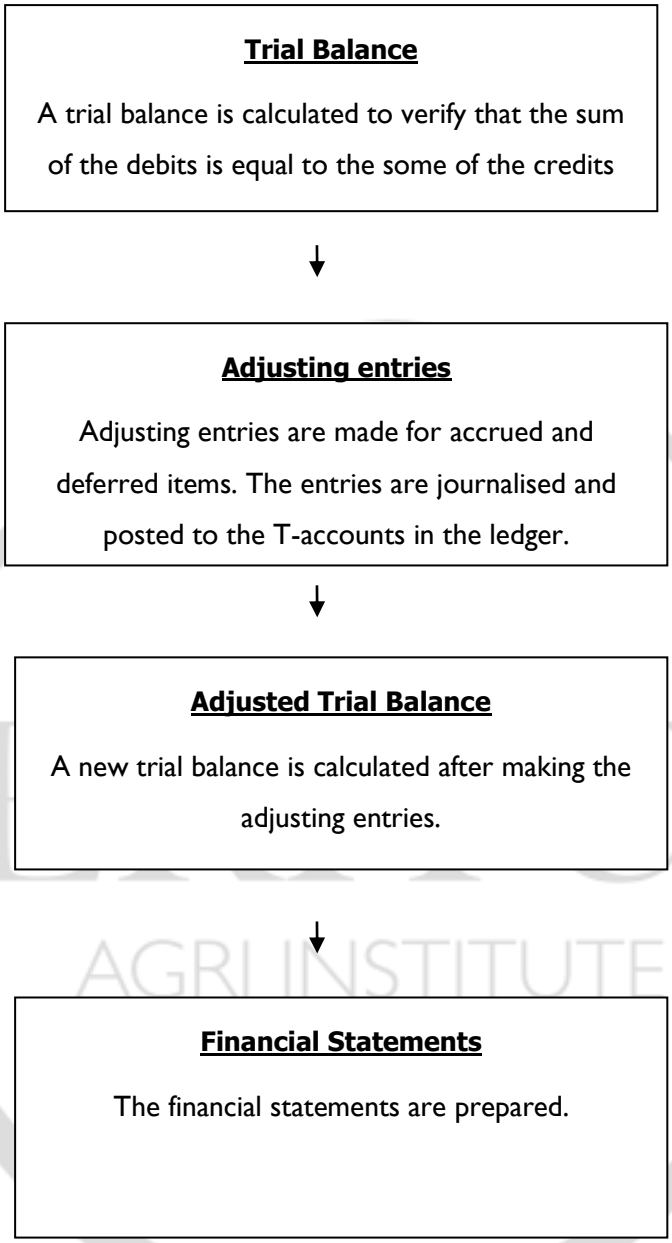
Accountants work through a cycle called the Accounting Cycle.

A manager does not need to go into great depth in terms of understanding the accounting behind the financials; however, it does help to have a basic idea of where the numbers are coming from.

The accounting profession is made up of various degrees of qualifications. You get accountants, chartered accountants, financial accountants, management accountants, auditors and bookkeepers.

The Accounting Cycle is illustrated:





Let us work through each step of the cycle:-

Step one:

Identify the Transaction
Identify the event as a transaction and generate the *source document.*

Transactions will include things like purchases, making a sale and issuing credit. Source documents are the documents that show proof of the transaction. Source documents include things like invoices, receipts, cheque counterfoils and credit notes.

Step two:

Analyze the Transaction
Determine the transaction amount, which accounts are affected and in which direction.

The transaction amount will be the amount in Rands of the purchase or sale etc. Accountants will then enter the transaction amount into the *accounts*. Accounts are the opened for each element of a financial statement and all the transaction amounts that apply to that particular item is recorded in the relevant account. Accounts are divided into two symmetrical sections called *debits* and *credits.* The debit section is shown on the left-hand side and the credit section is shown on the right hand side. The side on which the transaction is captured depends on whether the transaction increases or decreases the particular asset or liability or income or expense. For example, if there was increase in the asset account, the entry would be recorded on the debit side of the account. If the asset had decreased, the entry would be recorded on the credit side of the account.

Which side of the accounts need to be debited and which side of the accounts need to be credited can often be confusing to the non-accountant. For the purposes of this course, it is not essential for you to grasp accounting concepts in detail, however some examples of debits and credits are shown in the tables below:

Table depicting examples of debits and credits for expenses and incomes:

Debits (usually expenses)	***Credits (usually incomes)***
Cost of sales	Sales
Wages	Rent income
Water and electricity	Interest on a fixed deposit
Salaries	Discount received
Telephone	Bad debt recovered
Stationery	Provision of bad debt
Depreciation	Profit on sale of assets
Interest paid	Interest received

Table depicting examples of debits and credits for assets and liabilities:

Debits (usually assets)	***Credits (usually liabilities)***
Fixed assets such as land, buildings, vehicles and equipment	Non-current liabilities such as non-current loans.
Financial assets such as fixed deposits, fixed investments and any other deposits.	Current liabilities such as overdrafts and income received in advance.
Current assets such as inventory or prepaid expenses.	Dividends or distributions to members.

Accountants follow what is called "*The Double Entry Rule"*. An accounting equation must always balance, so a transaction always affects two elements at the same time. One element will be debited, and one element will be credited.

Let us look at a couple of very basic accounting examples:

Example 1

- On the 11th January 2009, wages were paid to an employee by the name of Andrew Smith in the amount of R500.

The wages account would be debited by R500 and the bank account would be credited by R500 at the same time, in line with the double entry rule.

Example 2

- On the 12th January 2009, the organisation received an amount of R350 from a customer for services that were rendered.

The bank account would be debited by R350 and the services rendered account would be credited by R350 at the same time in line with the double entry rule.

Step three:

Journal Entries

The transaction is recorded in the *journal* as a debit and a credit.

Once the accountant knows which accounts to debit and which accounts to credit, the records are made in the *books of first entry* or *Journals.* The examples used in Step two are shown as journal entries below:

THEMBA'S CLOTHING

General journal for January 2009

Date		Debit	Credit
11/01/2009	**Dr Wages**	**R500**	
	Cr Bank		**R500**
12/01/2009	**Dr Bank**	**R350**	
	Cr Services Rendered		**R350**

Step four

Post to Ledger

The journal entries are transferred to the appropriate T-accounts in the *ledger*.

The ledger is simply a summary of the transactions that were recorded in the journals. The example from Step two and Step three are shown in a simple ledger below. *CPJ* stands for "Cash Purchases Journal" and *CRJ* stands for "Cash Receipts Journal."

Dr. (+) **(Asset)** **(Wages)** **1** **(Cr. (-)**

Date	Details	Folio	R	Date	Details	Folio	R
11-01-2009	Cash	CPJ	500				

Dr. (+) **(Expense)** **(Bank)** **B1** **(Cr. (-)**

Date	Details	Folio	R	Date	Details	Folio	R
11-01-2009	Services rendered	CPJ	350	11-01-2009	Wages	CPJ	500
	Balance	c/d	150				
			500				**500**
				1-2-2009	2009 Balance	b/d	150

Dr. (+) **(Income)** **(Services Rendered)** **I2** **(Cr. (-)**

Date	Details	Folio	R	Date	Details	Folio	R
				11-01	2009 Bank	CRJ	350

Step five:

Trial Balance

A *trial balance* is calculated to verify that the sum of the debits is equal to the some of the credits

A trial balance is a list of all the balances of the accounts that were depicted in the general ledger. The trial balance can be used to confirm that the total of accounts with debit balances still equals the total of the accounts with credit balances. The example as carried over from Step four is shown below:

THEMBA'S CLOTHING

Trial balance as at 31 January 2009

	Folio	Debit R	Credit R
Balance Sheet Section			
Bank	B1		150
Income Statement Section			
Wages	I1	500	
Services Rendered	I2		350
		500	500

Step six:

Adjusting entries

Adjusting entries are made for accrued and deferred items. The entries are journalised and posted to the T-accounts in the ledger.

In accounting, adjustments are made at the end of the financial year to reflect any transactions that did not appear in the source documents. These adjustments are made for the sake of accuracy. Adjusting entries would need to be made in the journal and the ledger.

For example, let us say that wages was incorrectly entered at R500, and the correct amount was actually R450. Let us also assume that we owed R600 for the rent for the month but we paid R1 200 and this entry was not recorded. The adjustments would need to be reflected in the journal and ledger as follows:

THEMBA'S CLOTHING

General journal for January 2009

No	Detail	Debit	Credit
1.	Dr. Bank	500	
	Cr. Wages		500
	(Correction of error)		
2.	Dr. Pre-paid expense	600	
	Cr. Rent expense		600
	(Rent expense pre-paid)		

Dr. (+) (Expense) (Wages) I1 (Cr. (-)

Date	Details	Folio	R	Date	Details	Folio	R
11-01-2009	Bank		500	11-01	Bank		500
	Bank		450				

Now we put the correct entry through:

Dr. (+) (Cr. (-)

Date	Details	Folio	R	Date	Details	Folio	R
	Wages		450		Bank		450

Then we record the entry that was not entered:

Dr.(+) **(Cr. (-)**

Date	Details	Folio	R	Date	Details	Folio	R
	Rent paid		1 200		Bank		1 200

Dr. (+) **(Asset)** **(Pre-paid Expense)** **B2** **(Cr. (-)**

Date	Details	Folio	R	Date	Details	Folio	R
11-01-2009	Rent expense		600				

Dr. (+) **(Asset)** **(Bank)**

B1 **(Cr. (-)**

Date	Details	Folio	R	Date	Details	Folio	R
11-01-2009	Wages		500	11-01-2009	Wages		500
	Balance	b/d	1650		Wages		450
					Rent expenses		1200
			2150				2150
				01-02	Balance	b/d	1650

Dr. (+) **(Asset)** **(Rent Expense)** **I3**

(Cr. (-)

Date	Details	Folio	R	Date	Details	Folio	R

11-01-2009	Bank		1200	11-01-2007	Pre-paid expense		600
					Balance	b/f	600
			1200				1200
	Balance	b/d	600				

Step seven:

Adjusted Trial Balance

A *new trial balance* is calculated after making the adjusting entries.

Once the adjustments have been made, a new trial balance has to be drawn up. The example is depicted below:

THEMBA'S CLOTHING

Adjustment trial balance as at 31 January 2009

Details	Folio	Debit R	Credit R
Balance Sheet Account Section			
Bank	B1		1650
Pre-paid expense	B2	600	
Income Statement Section			
Wages	I2	450	
Rent expense	I3	600	
		1 650	**1 650**

Step eight:

Financial Statements
The financial statements are prepared.

The financial statements are then prepared from the trial balance.

Individual Formative Exercise 1A:

Complete Exercise 1A in your Portfolio of Evidence.

ACCOUNTING CONVENTIONS

An accounting convention is a common practice that is universally followed in recording and presenting accounting information. Accounting conventions help in comparing accounting data across business units. It is important for a manager to be aware of these accounting conventions when interpreting financial reports.

Some accounting conventions are explained in the table below:-

Accounting Convention	**Explanation**
Consistency	The same accounting principles should be used to prepare financial statements year after year. For example, depreciation on fixed assets is calculated using the same method each year.
Going concern	Accountants assume, unless there is evidence to the contrary, that a company is not going broke. This in turn has important implications for the valuation of assets and liabilities.
Prudence	Profits are not recognised until a sale is completed and costs are provided for in the accounts as soon as there is a reasonable chance that they may occur.
Realisation	Accounts recognise transactions at the point of sale or at the point of contract,

	rather than when payment is made.
Disclosure	Material and relevant facts are disclosed to interested parties such as shareholders or investors. Disclosure may be adequate, fair or full.
Objectivity	Accounting information is reported in a neutral way and is not biased.
Matching	Income should be properly "matched" with expenses in a given accounting period.

Individual Formative Exercise 1B:

Complete Exercise 1B in your Portfolio of Evidence.

Individual Formative Exercise 1C:

Complete Exercise 1C in your Portfolio of Evidence.

Learning Unit 2

The Financial Reports

Unit Standard	
116428	**Analyse and interpret the financial statements and physical records in an agri-business to generate managerial information**

Specific Outcomes

SO1: Use financial and physical information to compile financial statements and physical data to compile physical records

SO 2: Conduct a proper analysis of the financial statements and physical records of an agri business

Learning Outcomes

- Explain your financial reports with examples

  (Financial reports include audit reports, income statement, cash flow statement and balance sheet)

- Analyse financial statements and evaluate for authenticity and accuracy.

Critical Cross-field Outcomes

Organise	Demonstrating
Collecting	Contributing
Science	Identifying

INTRODUCTION

The previous Learning Unit demonstrated on a basic level how financial reports are arrived at. As a manager, you probably will not get too involved in the accounting behind these reports; however, you will be required to analyse the reports in order to make decisions. In this section, the elements of the financial reports will be explained in more detail.

Different forms of business will have different legal requirements in terms of financial reports:

Sole proprietorship (1 owner): *- The owner has a personal interest in the business.*

- Not necessary to publish and audit financial statements.

Partnerships (2-20 partners) *- Partners have a personal and unlimited liability for the debts of the partnership.*

- It is not necessary to publish and audit the financial statements.

Private Company (1-50 directors /shareholders – Pty Ltd) *- Financial statements must be drawn up and must be audited by a chartered accountant, but not compulsory to publish.*

Public Company (+7 people – Ltd) *- Financial statements must be drawn up and must be audited by a chartered accountant.*

- Compulsory disclosure of statements and constitution of the company's affairs.

Close Corporation (1-10 members – CC) *- Financial statements are only required in simplified form, not necessary to audit or publish.*

THE BALANCE SHEET

A Balance Sheet is very simply a statement of position at a given date. It is like a snapshot at the close of a given day of a business.

The balance sheet lists all the money owned or owed to a business (the *assets*) and the money owed by a business (the *liabilities*).

The balance sheet also includes what the owners or shareholders have put into the business (in other words the source of financing of the business) otherwise known as the shareholder's *equity*.

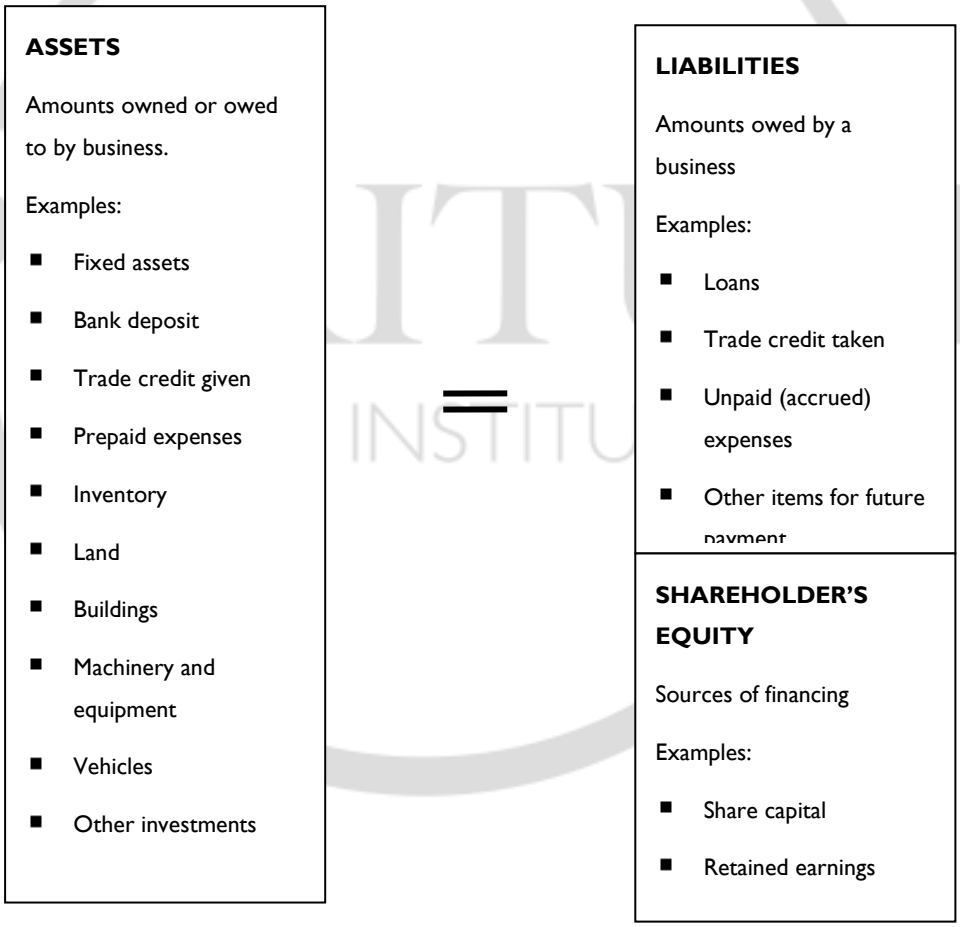
The Balance Sheet is stated as:

ASSETS = LIABILITIES + EQUITY

Basically, you would always want to own more than you owe. So, your assets should therefore not only be equal to any liabilities (loans you may have taken), but also the actual financing that you have put into the business. Arithmetically, shareholders equity always equals assets less liabilities.

The diagram below illustrates what goes into a balance sheet:

The Balance Sheet: Assets equal Liabilities plus Shareholder's Equity



Some Balance Sheets that you come across will be laid out vertically and others horizontally.

Assets and liabilities may be classified as short term (current assets and current liabilities) or as long term. Short term assets and liabilities are usually those recorded as of the current financial year, whereas long term assets and liabilities are those that are recorded over periods greater than one year.

Let's look at the example Balance Sheet below:

Summer Trading (Pty) Ltd

Balance Sheet as at 30 September 2009

	2004	**2005**
ASSETS	**R**	**R**
Current assets		
Cash at bank	10 000	10 000
Accounts receivable	35 000	30 000
Inventory	25 000	20 000
Total current assets	**70 000**	**60 000**
Fixed assets		
Plant and machinery	20 000	20 000
less depreciation	(12 000)	(10 000)
Land	8 000	8 000
Intangible assets	2 000	1 500
Total non-current assets	**18 000**	**19 500**
TOTAL ASSETS	**88 000**	**79 500**
LIABILITIES		
Current liabilities		
Accounts payable	20 000	15 500
Taxes payable	5 000	4 000
Total current liabilities	**25 000**	**19 500**

Non-current liabilities

Loans – non current	15 000	10 000
Total non-current liabilities	**15 000**	**10 000**
TOTAL LIABILITIES	**40 000**	**29 500**

SHAREHOLDER'S EQUITY

Paid-in share capital	40 000	40 000
Retained earnings	8 000	10 000
TOTAL SHAREHOLDER'S EQUITY	**48 000**	**50 000**
LIABILITIES AND SHAREHOLDER'S EQUITY	**88 000**	**79 500**

As you can see, total liabilities and shareholder's equity equal total assets.

Let's break down some of the main elements that could show on a Balance Sheet:-

Element of Balance Sheet	**Explanation**
ASSETS	Economic resources that are expected to produce economic benefits for their owners. Assets can be buildings, vehicles or machinery, but they can also be patents or copyrights that provide financial advantages.
Current assets	Assets that are usually converted to cash within a period of one year. Creditors will closely monitor a firm's current assets.
Types of current assets:-	
Cash and bank deposits	These are the most liquid of current assets and are used to pay the bills. (Transactional bank deposits are also regarded as cash).
Cash equivalents	These are not cash but can be converted into cash easily. (examples include securities and money market funds)
Accounts receivable	Money that should be collected from customers. Due to the fact that a lot of business is done on credit, this item often forms a significant part of the balance sheet. The notes to the accounts should show long

	standing debts and provision should be made for any bad debts. A manager should check to see if receivables are growing more quickly than sales, as this could mean trouble for the organisation.
Inventory	Inventory includes raw materials, part completed items in progress and completed items. A manufacturing entity will have all three types of inventory, while a retail entity will only have completed items. The extent to which inventory can be turned into cash will vary and the underlying true market value of inventory will also vary. The notes to the accounts will provide more information in this regard. A manager should watch to see that inventory is not growing faster than sales as this could mean a slowdown in sales.
Pre-paid expenses	These are amounts paid in advance (for example rentals). Pre-paid expenses can be difficult to turn back into cash, but as long as the business is operating, they are considered a measure of stored value.
Long –term assets	Assets with a life of greater than one year.
Types of long-term assets:-	
Fixed assets	Tangible assets with a useful life greater than one year. Examples include buildings, property, equipment, machinery, production plants and vehicles. These are valued at total acquisition cost. Fixed assets are important as they represent long term, illiquid investments. The notes to the accounts may give more detail on the fixed assets.
Depreciation or accumulated depreciation	The process of allocating the original purchase price of a fixed asset over the course of its useful life. Depreciation appears as a deduction from the original value of the fixed assets. There are different ways in which depreciation can be calculated and the manager should be aware which method has been used. (Refer to section 4.3.1.1 for more detail on depreciation).
Intangible assets	Non-physical assets such as copyrights, franchises and patents. It is not always easy to estimate the value of an intangible asset. For some organisations, an intangible asset can prove to be very valuable. Intangible assets are usually shown separately from tangible assets or

	fixed assets.
LIABILITIES	Obligations that an organisation owes to outside parties or the rights of others to the services or money of the organisation. Examples include debts to suppliers, debts to employees and bank loans.
Current liabilities	Those obligations that are usually paid within the year.
Types of current liabilities:-	
Accounts payable	Debts owed to suppliers for the purchase of goods and services. (for example, the company buys their goods "on account")
Taxes payable	Any taxes that are payable in accordance with the legislation.
Short-term loans	Borrowings from banks or other lenders that are repayable within 12 months.
Non-current liabilities	A debt owed over a period greater than one year, often paid in instalments. The portion to be paid off in the current year is considered a current liability.
Types of non-current liabilities: -	
Non-current loans	More structured kind of borrowing over periods greater than one year. Managers should note that long –term borrowing should be matched against long-term assets. If short-term borrowing is being used to finance long-term assets the organisation could be in trouble.
Provisions	Balance sheets may also include provisions or contingent liabilities (probable future costs or losses where the timing is not certain)
SHAREHOLDER'S EQUITY	The value of a business to its owners after all of its obligations has been met. This is generally reflected by the amount of capital invested by the owners, as well as any profits re-invested.
Share capital	The book value of money raised by issuing equity or shares.
Treasury stock	The company's holding of its own stock repurchased in the open market.
Retained earnings	The reinvested profits or profits not distributed as dividends. Net profit

	less dividends is equal to retained earnings.

Now that you have an understanding of Balance Sheets, let's do some activities using other examples of Balance Sheets. Ultimately, you will need to be able to interpret your own organisation's Balance Sheets.

Individual Formative Exercise 2A: Complete Exercise 2A in your Portfolio of Evidence.

If you understand the concept of a Balance Sheet, you should not be afraid to tackle any kind of Balance Sheet, no matter how complicated. The layout of a Balance Sheet will follow the same basic structure, but there may be some things in the balance sheet you have not come across before. If this is the case, then do not be afraid to research the things you do not understand. The following exercise will help to build your confidence.

Individual Formative Exercise 2B: Complete Exercise 2B in your Portfolio of Evidence

THE INCOME STATEMENT

The Income Statement (or Profit and Loss Account) is basically the organisation's record of sales and costs for a specified period of time, usually a month, quarter or year. The bottom line of the Income Statement is the bottom line of the organisation or the net profit or loss.

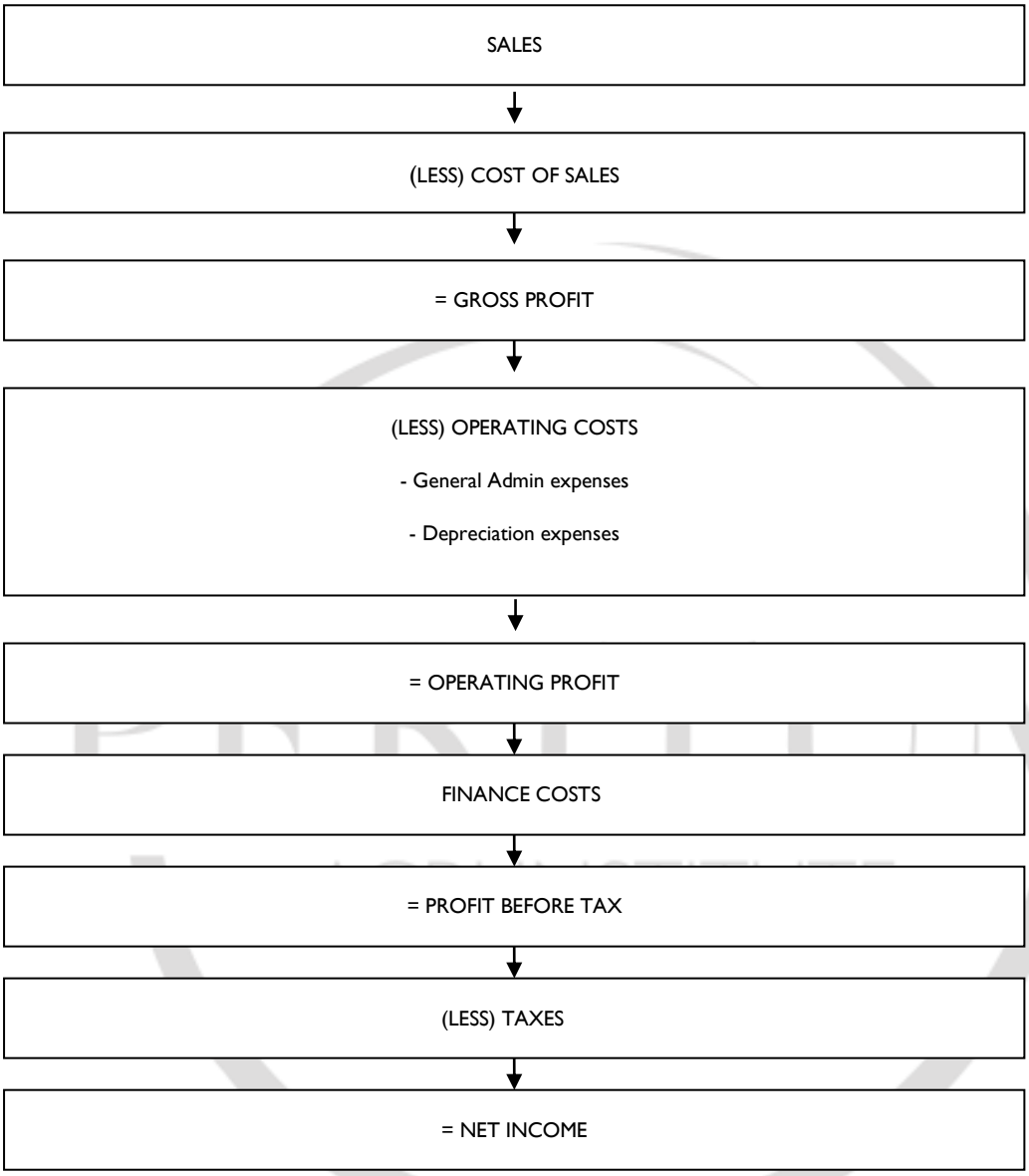
The simplest equation to describe an Income Statement is:

NET INCOME = REVENUE - EXPENSES

The Income Statement does not include capital transactions such as investments, fixed assets or borrowing; however, it does include the current income and costs related to these items such as interest and depreciation.

The diagram below illustrates a typical format of an Income Statement:

A typical format of an Income Statement



Income statements are not all structured exactly the same; however, this example is a typical income statement format.

Arial Trading (Pty) Ltd

Income Statement for the year ended 28 Feb 2009

	R
Income	
Sales	4076
(Less) Cost of Sales	
Cost of goods sold	(3207)
Gross Profit	**869**
Operating Expenses	
General Admin/ business expenses	(227)
Depreciation expenses	(206)
Total Operating Expenses	**(433)**
Operating Income	**436**
Interest expense	(3)
Earnings before Taxes	**433**
Taxes	165
Net Income	**268**

Let's break down some of the main elements that could reflect on an Income Statement:-

Element of Income Statement	**Explanation**
INCOME	The revenue or turnover of an organisation. The inflows form the delivery or manufacture of a product or from the rendering of a service. Income could be reflected by Sales, Revenue or Turnover.
Sales (also sometimes referred to as Revenue or Turnover)	Money derived from selling the company's product or service.

Cost of Sales	Includes all the spending *directly* associated with sales. For wholesalers and retailers, the cost of sales is essentially the prices paid to acquire the services or goods that will be resold. For manufacturers, the cost of sales is all the spending directly attributed to production (examples would include raw materials, factory overheads and wages). Cost of sales could also include research and development costs. By implication, cost of sales requires a calculation – these calculations could vary from very simple to more complexes, depending on the type of organisation. An example of a Cost of Sales calculation would be *Opening Stock + Cost of Purchasing – Closing Stock.* It is important to note that cost of sales does not include *indirect* costs (costs which cannot be directly attributed to sales and production.) Cost of sales is always a contra-entry as it is a cost.
GROSS PROFIT	The Sales less the Cost of Sales is the Gross Profit.
Operating Expenses	All expenses related to administering the business and marketing and distributing the product or service. Operating expenses might include rent, telephone, wages, water and electricity, advertising, bank charges, insurance, fees, salaries, repairs and other expenses. Depreciation is also considered an operating expense.
OPERATING PROFIT (OR LOSS)	Operating profit (or loss) is calculated by deducting Operating Expenses from Gross Profit. This is the core of the income statement.
Finance costs / Revenue or expenses / Gains or losses that are not part of the company's normal operations.	These could include interest paid on loans, interest received on deposits and investment income.
PROFIT (OR LOSS) BEFORE TAX	This is the operating profit (or loss) less the other revenue / expenses or gains or losses that are not part of the company's normal operations.
Tax	Company taxes that are payable in accordance with legislation. These would be based on taxable income.
NET PROFIT (OR LOSS)	This is the net income (or loss) after tax.

Now that you have an understanding of Income Statements, let's do some activities using other examples of Income Statements. Ultimately, you will need to be able to interpret your own organisation's Income Statements.

Individual Formative Exercise 2C:

Complete Exercise 2C in your Portfolio of Evidence.

If you understand the concept of an Income Statement, you should not be afraid to tackle any kind of Income Statement, no matter how complicated. The layout of an Income Statement will follow the same basic structure (with some variations), but there may be some things in the Income Statement you have not come across before. If this is the case, then do not be afraid to research the things you do not understand. The following exercise will help to build your confidence.

Individual Formative Exercise 2D

Complete Exercise 2D in your Portfolio of Evidence

THE CASH FLOW STATEMENT

The Cash Flow Statement is the third major financial statement. Income Statements show the passing relationship between money that changes hands, while the Cash Flow Statement focuses on changes in liquidity.

A Cash Flow states the sources and uses of funds and explains the changes in cash and cash equivalents over the period of accounts.

Cash flows can be done using one of two methods, the *direct* method or the *indirect* method. Cash flow statements do not include the amounts recorded from credit, only cash.

Let us start by explaining the *direct* method.

The direct method directly shows receipts from customers and payments to suppliers. Let's look at the example below:

Autumn Consulting (Pty) Ltd

Cash flow statement

	R
Cash receipts from sales	400 000
Cash paid to suppliers	(80 000)
Cash paid to employees	(100 000)
Cash generated from operations	**220 000**
Interest paid	(1 000)
Interest earned	2 000
Taxes	(20 000)
Net cash generated by operating activities	**201 000**

The direct method cash flow statement is fairly self-explanatory.

Let us now look at the *indirect* method. This method is used more frequently and is essentially the reversing out of non-cash items from the net-income. The indirect cash flow looks at the three components by which cash enters and leaves a company: core operations, investing and financing.

Operations

The operations component reflects how much cash is generated from a company's products or services. Generally speaking, changes made in cash, accounts receivable (receipts from customers), depreciation, inventory, and accounts payable (payments to suppliers and employees) are reflected in cash from operations.

Cash flow is calculated by making certain adjustments to net income, by subtracting or adding differences in revenue, expenses and credit transactions resulting from transactions that occur from one period to the next. These adjustments are made because non-cash items are calculated into net income (from the Income Statement) and total assets and liabilities (from the Balance Sheet).

Examples of adjustments that are made to calculate cash flow are:-

- Depreciation is added back into net income. (This is because depreciation is an amount deducted from the total value of an asset and is not an actual cash expense).

- Changes in accounts receivable from one accounting period to the next are reflected. If accounts receivable decreases, then more cash have entered the company (more people are paying off their credit) and this is then added to net income. If accounts receivable increases then this is deducted from net income, because these amounts are not cash, but credit.
- If inventory increases, then more money has been spent. If the inventory was purchased with cash, the increase in the value of the inventory is deducted from net income. A decrease in inventory implies that more cash is coming in and this would therefore be added to net income. However, if inventory was purchased on credit, an increase in accounts payable would occur and the increase in the amount would be added to income.
- The same logic holds true for salaries payable, taxes payable and prepaid insurance. If something has been paid off, then the difference in the value owed from one year to the next has to be deducted from the net income. If there is an amount that is still owed, then this would reflect as in increase in the net income.

Investing

Investing activities are the acquisition and disposal of long-term assets and investments not included in cash equivalents. When an investment is made, this is a "cash-out" item. When a company divests of an asset, this is a "cash-in" item.

Financing

Financing activities result changes in the size and composition of the organisation's capital and borrowings.

Examples: When capital is raised, this is a "cash-in" item. When dividends are paid, this is a "cash-out" item. If bonds are issued, this is a "cash-in" item, however if interest is paid to bondholders, this is a "cash-out" item.

The following example is an illustration of a basic cash flow statement using the indirect method:

Jackson's trading

Cash Flow Statement for the year ended 31 December 2008

Cash flow from operations

Net earnings	2 000 000

Additions to cash	
Depreciation	10 000
Decrease in Accounts Receivable	15 000
Increase in Accounts Payable	15 000
Increase in Taxes Payable	2 000
Subtractions from cash	
Increase in Inventory	(30 000)
Net Cash from Operations	2 012 000
Cash flow from investing	
Equipment	(500 000)
Cash flow from financing	
Interest payable	10 000
CASH FLOW FOR THE YR ENDED 31 DEC 2005	**1 522 000**

Looking at the above example, the bulk of the positive cash flow stemmed from operations, which is a good sign for investors as this means there is enough money to buy new inventory. The purchasing of new equipment is a sign of growth, while cash is available to pay any loans.

Not all cash flow statements exhibit a positive cash flow. However, a negative cash flow is not always a bad thing if, for example, the company is undergoing an expansion.

The Cash Flow statement reveals the following:-

- Cash earnings
- How the organisation is utilising its funds
- How the organisation is being financed
- Any need for outside financing
- The ability of the organisation to obtain outside financing
- The organisation's investments or divestments

- The organisation's ability to generate future cash flows.
- The organisation's ability to meet its obligations

In summary, cash flow is essentially a key indicator of the organisation's health.

Now that you have an understanding of Cash Flow Statements, let's do some activities using other examples of Cash Flow Statements. Ultimately, you will need to be able to interpret your own organisation's Cash Flow Statements.

Individual Formative Exercise 2E

Complete Exercise 2E in your Portfolio of Evidence.

If you understand the concept of a Cash Flow Statement, you should not be afraid to tackle any kind of Cash Flow Statement, no matter how complicated.

The layout of a Cash Flow Statement will follow the same basic structure (with some variations), but there may be some things in the Cash Flow Statement that you have not come across before.

If this is the case, then do not be afraid to research the things you do not understand. The following exercise will help to build your confidence.

Individual Formative Exercise 2F

Complete Exercise 2F in your Portfolio of Evidence

THE AUDIT REPORTS

An audit checks to see that the financial statements comply with reporting regulations are a true view of the state of affairs in the organisation. An auditor will also look out for any suspicious activity such as possible fraud. If an auditor finds something that appears is not being done properly, he or she will *qualify* the report with suitable warnings? If a report has been qualified by an auditor, it generally means that the organisation has to deal with the issues raised.

Learning Unit 3

Interpreting Financial Statements

Unit Standard	
116428	**Analyse and interpret the financial statements and physical records of an agri-business to generate managerial information**

Specific Outcomes

SO2: Conduct a proper analysis of the financial statements and physical records of an agri-business

Learning Outcomes

- Apply ratios to measure the profitability and liquidity of an entity.
- Apply ratios to measure the working capital and asset utilisation of an entity.
- Apply ratios to measure the return of an entity.

  (Return ratios include return on equity, return on investment and debt ratio)
- Make recommendations regarding the profitability of, liquidity, working capital, and return and resource utilisation by using results obtained from ratio application.

Critical Cross-field Outcomes

Organise	Demonstrating
Collecting	Contributing
Science	Identifying

INTRODUCTION

Financial reports were introduced in the previous Learning Unit, along with explanations of each type of report. These financial reports provide information about an entity's financial position and its financial performance over a period of time. However, it also important for managers to measure long term trends of performance.

The four key areas of measurement are:-

- Efficiency: looks at how effectively an organisation is managing its assets
- Liquidity: a measure of how easily an organisation's assets can be converted to cash.
- Profitability: provides information about whether the performance is improving or getting worse.
- Capital structure: provides information about whether owner's capital or borrowed capital is used.

Financial analysis and interpretation are used to help with these areas of measurement. Ratio analysis is a form of financial analysis. A ratio measures the relationship between two measurable items by dividing one item by another item. A financial ratio is a relationship between two different items from the financial statements and can consist of figures taken from both the Balance Sheet and the Income Statement. Once the ratios are calculated, they can then be interpreted.

Ratio analysis can assist a manager to answer questions such as:-

- How effectively are assets being managed?
- How much debt can we afford?
- Is there enough cash to pay short –term debts?

Ratios can be used not only to analyse one's own financial reports, but also published competitor's and other relevant companies published reports. The following sections explain the different ratios.

LIQUIDITY

- All current assets are eventually transformed into cash.
- All current liabilities eventually have to be paid for with cash.
- The source for the cash, which is needed to pay current liabilities, is current assets.
- There should, therefore, always be sufficient current assets in relation to the level of current liabilities.
- This relationship is called liquidity.
- Liquidity refers to a firm's ability to meet its obligations over the short term, that is to have the necessary cash available from cash reserves, cash sales and debtors to pay for sufficient stock and to make payments due to creditors.

Two ratios are commonly used to measure a firm's liquidity:

Current ratio = current assets ÷ current liabilities

= 24 000 000 ÷ 10 000 000

= 2.4 times

It gives an indication of the company's ability to meet its obligations over the short term.

(also called the liquidity ratio). Jimco (pg 52) had R2.40 in current assets for every R1.00 in current liabilities. This compares favourably with the rule of thumb of 2 for the current ratio.

Quick ratio = (current assets - stock) ÷ current liabilities

= (24 000 000 - 12 000 000) ÷ 10 000 000

= 1.2 times

This is a stricter measure of liquidity than the current ratio and is also known as the acid test. Stock first has to be sold before it can be converted to cash. Jimco had R1.20 of cash or cash claims (debtors) for every R1.00 in current liabilities. This does compare favourably with the rule of thumb of 1 for the quick ratio. Investment in stock may be too high if this ratio is out of line, especially if the previous one was in line.

ACTIVITY RATIOS

These ratios measure the speed (in terms of days or number of times per year) current assets and liabilities are converted into cash. Activity ratios are, therefore, also a measure of a firm's liquidity.

There are two good reasons for selling stock as quickly as possible:

- There are costs involved in keeping stock on hand e.g. storage costs, insurance costs and the cost of the funds tied up in capital.
- The only way to get the gross profit connected to stock is to sell it.

The speed at which stock is sold is measured by the stock turnover ratio.

Stock turnover ratio = cost of goods sold ÷ stock

= R38 000 000 ÷ R12 000 000

= 3.17 times per year

This means that Jimco managed to sell the average amount of stock 3.17 times per year.

Industry average 2.9 times.

It is customary to express stock turnover in days. This is done by dividing 365 by the stock turnover ratio just calculated.

Stock turnover in days = 365 ÷ 3.17

= Therefore, it takes 115 days, on average, to sell Jimco's stock.

Cross sectional or time series analysis is needed to judge this figure. Stock turnovers vary widely over different industries e.g. greengrocers vs. jewellers.

Similarly, to stock, the quicker debtors are collected, the better for the company.

Debtor's collection ratio

= credit sales ÷ debtors

= R51 000 000 ÷ R10 000 000

= 5.1 times

This means that Jimco, on average, collects its outstanding short-term debt 5,1 times per year.

It is more customary to express this ratio in days per year.

Debtor's collection period in days

= 365 ÷ 5.1

= 71.57 days

Jimco takes, on average, 71.57 days to collect its outstanding short-term debt (debtors). Cross sectional or time series analysis is needed to judge this figure. If Jimco extended credit terms of 30 days to its customers, a collection period of 71.57 days would be reason for concern.

The speed with which the company pays its creditors is measured by the following ratio. The object here is to pay slower rather than quicker, but to take care not to jeopardise the relationship with the suppliers.

Creditors payment ratio:

= credit purchases ÷ creditors

= R38 000 000 ÷ R3 000 000

= 12.67 times

Once again, it is more customary to express this ratio in days:

Creditors' payment period:

= 365 ÷ 12.67

= 28.81 days

Jimco takes, on average, 28.81 days to pay its creditors. Cross sectional or time series analysis is needed to judge this figure. If Jimco received credit terms of 30 days from its suppliers, a payment period of 28.81 days may indicate that they have no difficulty in meeting creditor payments on time.

DEBT

A company's ability to survive over the long term, which is related to its debt position, is measured by these ratios. The debt position of a firm indicates the amount of non-owner or non-self-generated funds used by the business. In general, the more debt a firm uses, the higher the risk. Due to the influence of financial leverage, debt could actually increase the return to owners but at the risk of possible insolvency should the company not be able to service its debt.

Debt is part and parcel of the way modern companies are financed. As long as the margin of safety provided by own funds is sufficient and the company is able to meet its debt related obligations there is no reason why debt could not be used. The only proviso is that the company must earn more with the borrowed funds than the cost thereof, that is positive financial leverage.

Debt ratio:

= total liabilities ÷ total assets

= R10 000 000 + R10 700 000 ÷ R31 000 000

= 0.668 or 66.8%

Industry norm 47.7%

For each rand invested in assets, R0.66 was financed by debt. Put differently, only

R0.34 of each rand invested in assets belongs to the owners.

Times interest earned (TIE)

= Earnings before interest and tax ÷ Interest paid

= R4 000 000 ÷ R1 000 000

= 4 times

Interest is covered more than the 2.4 times of the industry norm. Note that when TIE equals 1, net income before tax is zero. Although Jimco's debt ratio seems to be on the high side, interest payments should easily be met, as indicated by the TIE ratio.

Note that these two ratios should always be judged together. A company can have a fairly low (good) debt ratio and a very low (bad) TIE ratio and vice versa.

PROFITABILITY

Companies need profits to survive, without profits there would not be much point in continuing to trade. Profits vary from year to year, even from season to season. If there is a trend in profits, especially downwards, this should be identified as soon as possible.

MARK- UP PERCENTAGE

$$\text{Mark-up percentage} = \frac{\text{Gross Profit}}{\text{Cost of Sales}} \times 100$$

How to interpret this ratio:

An organisation may price a product by adding a percentage to the cost of the product. Gross profit is the amount resulting from mark-ups on sold products. If the mark-up is very low, this could be an indication that products are being sold too cheaply. This ratio can also be used to ascertain if there was any increase in mark-ups from one period to the next.

Gross profit margin = gross profit ÷ sales

= R13 000 000 ÷ R51 000 000

= 25.49 %

Industry margin – 27.5 %

The gross margin indicates what portion of the selling price remains after paying for the stock. This is directly related to the pricing policy of either the company itself or its suppliers.

Increasing selling prices would increase the gross margin but could lead to less units being sold because of price competition. Buying for less will also increase the gross margin, but care should be taken to buy stock of the right quality.

Operating profit margin = operating income ÷ sales

= R4 000 000 ÷ R51 000 000

=7.84 %

Industry average – 8.1 %

The operating margin indicates what portion of the selling price remains to pay for interest, taxes and dividends. It is also an indication of the control over operating expenses. A decreasing operating profit margin while the gross profit margin stays constant, points to an increasing in overheads without a comparison increase in prices and/or volumes.

Net income margin = net income ÷ sales

= R1 800 000 ÷ R51 000 000

= 3.53 %

Industry Ave – 3 %

The net profit margin measures the percentage remaining from each sales rand after all expenses have been paid. This ratio is often cited as the measure of a company's success. What constitutes a "good" net profit margin differs considerably across industries. A far better measure of success is to relate the net income to funds invested, as in the next ratio.

Return on Assets {ROA)

= Profit before interest and tax ÷ total assets

= R 4 000 000 ÷ R31 000 000

= 12.9%

Industry Ave – 4.2 %

Also called the return on investment, this ratio measures the overall effectiveness of management in generating after tax profits with available assets (which equals total capital). The owners of Jimco will have to decide whether this is acceptable or not. If an increasing trend in ROA can be identified, this fairly low percentage may be quite acceptable.

The Return on owner's equity (ROE) ratio is also calculated from the information given in the Income and Expenditure statement. This ratio is of the net income of the business after it has paid tax, to the owner's equity; (Being the amount of money which the owner or owners have invested in the business, also known as net worth of the business). It calculates the return which the owner is getting on the money he invested in the business. The results of this ratio are also shown as a percentage. It is calculated as follow:

$$\frac{\text{Net income after tax}}{\text{Owners' equity}} \times 100$$

The debt Equity Ratio shows how a business is financed; in other words how much money the business is lending in comparison to the amount of money that the owner has invested in the business.

It is important to note that if a business has borrowed more than what the owner has invested, it might be a sign that the owner lacks confidence in the business, or that he does not have the funds necessary to expand the business.

Debt Equity Ratio:

$$\frac{\text{TOTAL DEBT}}{\text{TOTAL EQUITY}}$$

This calculation requires the TOTAL Debt, that is, short term and long-term debt. It also requires the TOTAL equity that is the full amount that the owner has invested in the business together with any profits that the business has made since the business opened.

Total Asset Turnover

$$\text{Total Asset Turnover} = \frac{\text{Sales}}{\text{Total Assets}}$$

How to interpret this ratio:

The bigger the ratio, the more efficient the utilisation of total assets.

Fixed Asset Turnover

Fixed Asset Turnover = Sales

Fixed Assets

How to interpret this ratio:

Fixed assets such as equipment or property require a large investment and it is important that the sales justify this investment. The bigger the ratio, the more efficient the utilisation of fixed assets.

Jimco Ltd. BALANCE SHEET December 31, 1981(Thousands of Rands)

ASSETS

Current assets:		
Cash		1 400
Marketable securities (market value, R320)		300
Account receivable		10 000
Inventories		12 000
Prepaid expenses		300
Total current assets		24 000
Fixed assets:		
Land		2 000
Plant and equipment	12 300	
Less: Accumulated depreciation	7 300	
Net plant and equipment		5 000
Total fixed assets		7 000
Total assets		31 000
LIABILITIES AND OWNER'S EQUITY		
Current liabilities:		
Accounts payable		3 000
Notes payable, 9%, March 1, 1982		3 400
Accrued salaries, wages and other expenses		3 100

Current portion of long-term debt	500
Total current liabilities	10 000
Non-current liabilities:	
Deferred income taxes	1 500
First mortgage bond, 7%, due January 1, 1995	6 300
Debentures, 8.5%, due June 30, 1991	2 900
Total long-term liabilities	10 700
Owner's equity	
Common stock (par value R 1.00)	100
Additional paid-in capital	2 000
Retained earnings	8 200
Total owner's equity	10 300
Total liabilities and owner's equity	31 000

Income statement for Jimco for the year ended December 31, 1981(Thousands of Rands)

Net sales		51 000
Cost of goods sold		(38 000)
Gross profit		13 000
Operating expenses		

Selling expense	3 100	
Depreciation expense	500	
General and administrative expense	5 400	(9 000)
Net Operating Income (NOI)		4 000
Interest expense		(1 000)
Earnings before taxes (EBT)		3 000
Income taxes		(1 200)
Net income (NI)		1 800

Individual Formative Exercise 3A

Complete Exercise 3A in your PoE.

DIFFERENCE BETWEEN FIXED AND VARIABLE COST

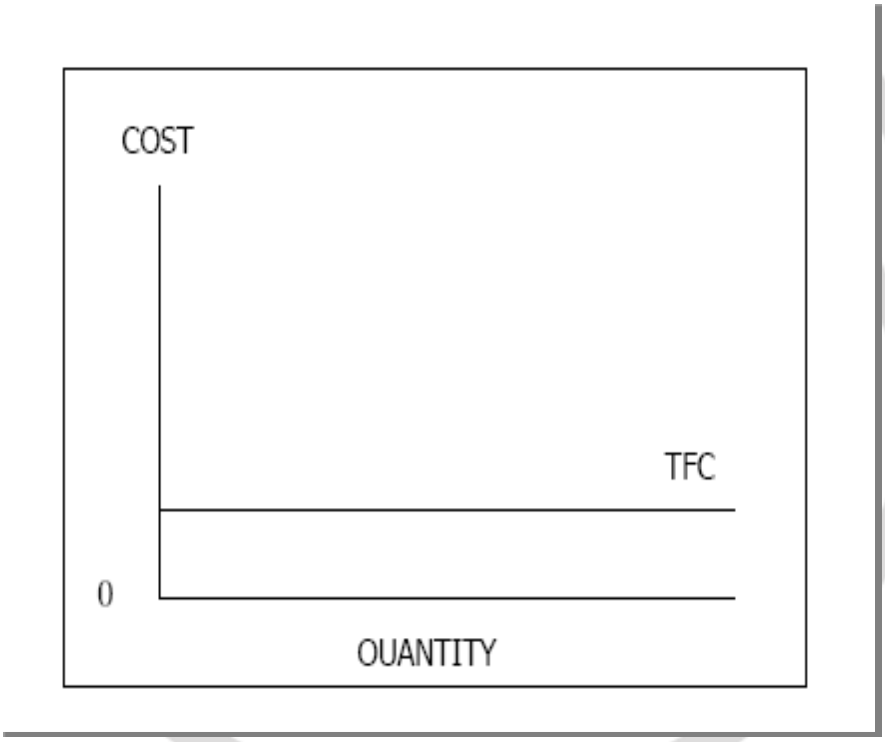
Fixed Cost

Fixed costs are those cost that will stay fairly constant within a period of time like a month or a year even if no production or sales are generated.

Typical fixed costs are rent of building or interest on a bond, salaries of managers, hire of equipment and depreciation on equipment.

The rent of the office will stay the same, even if no clients are seen or no income is generated.

The behaviour of fixed cost could be illustrated as follows:



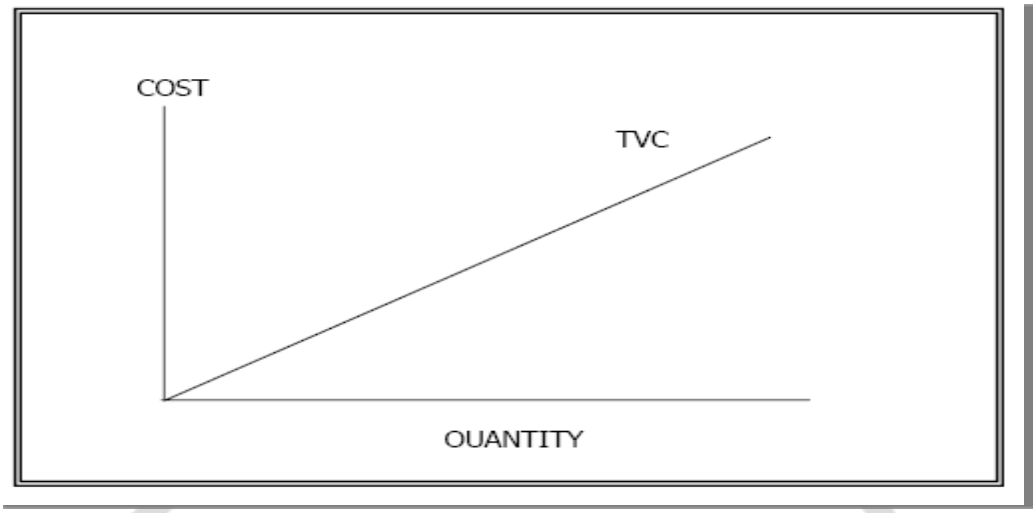
Although fixed cost is fixed even if no sales are done, fixed cost per unit decreases with an increase in production.

Variable Cost

The total variable cost will vary (increase or decrease) proportionally with the quantity manufactured or used.

Typical variable costs are materials, direct labour, water and electricity, etc.

The behaviour of total variable cost could be illustrated as follows:

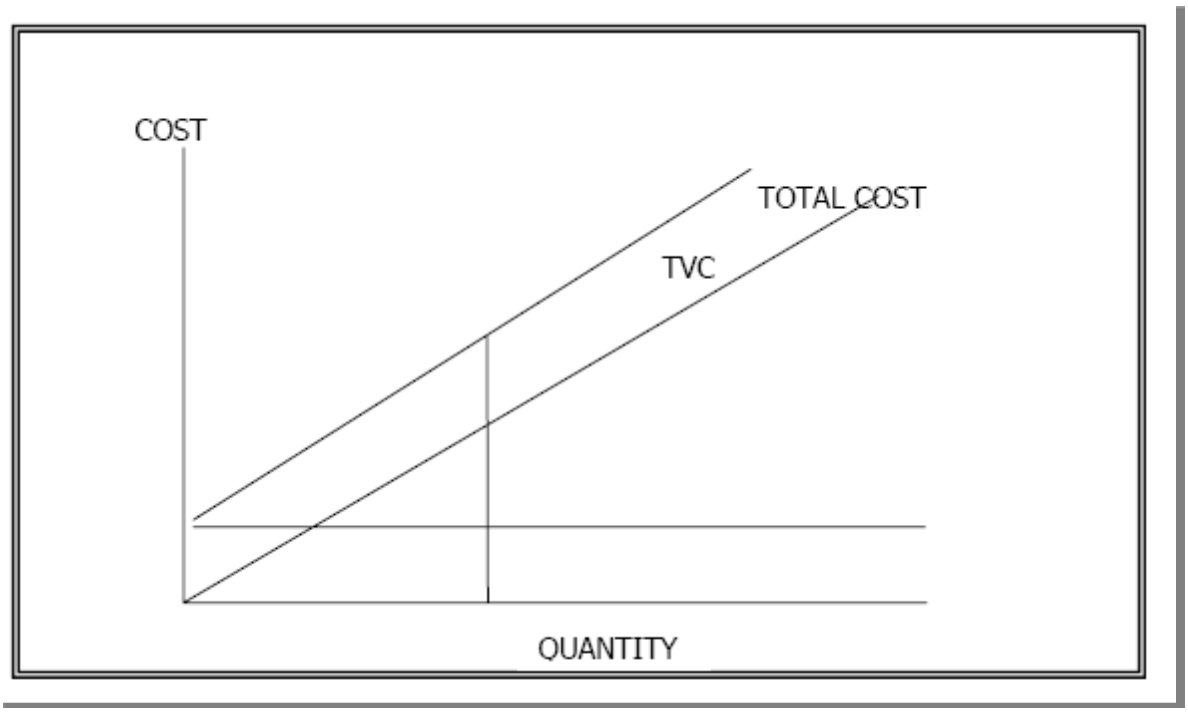


Although total variable cost varies if production increases or decreases, the variable cost per unit stays the same for different levels of manufacturing or sales.

Total Cost

Total cost is simply the fixed cost and variable cost added together.

The behaviour of total cost could be illustrated as follows:



Note that at zero production the variable cost is zero, but the fixed cost must stil be paid. As production increases, the variable cost must be added to the fixed cost to give you the total cost.

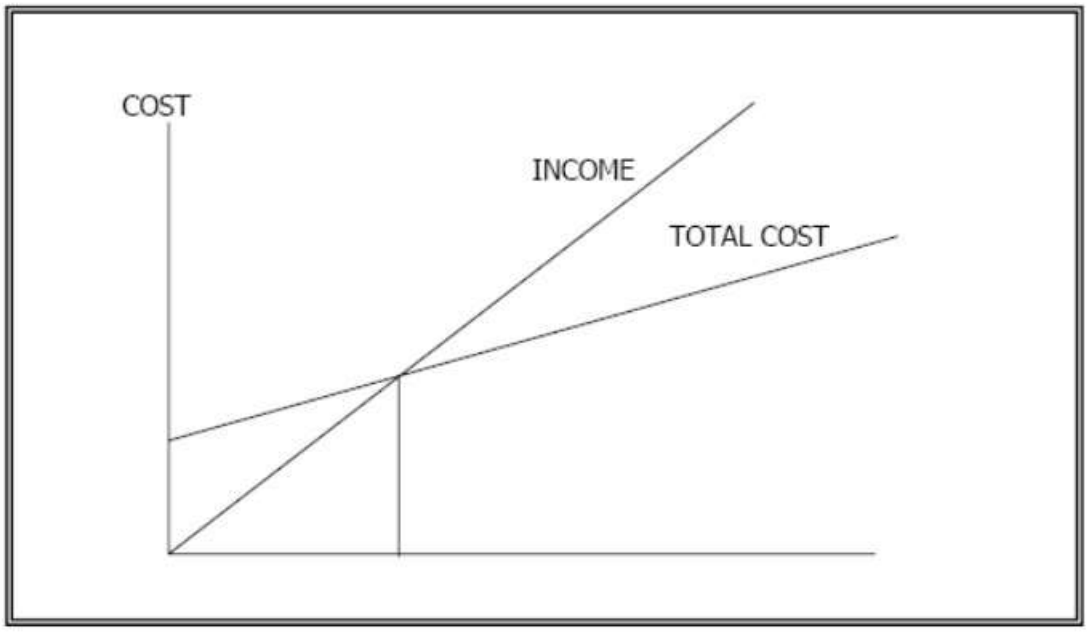
BREAK-EVEN ANALYSIS

The break-even point is that point in the volume of manufacture and sales where the total income derived from the sales of the product equals the total costs. No profit or loss is made at this specific volume.

When an entrepreneur wishes to start a new business, he will try his best to increase the production and sales volume to such a level that the break-even point can be passed as soon as possible. It is interesting to note that once the break-even point is passed, companies with a high fixed cost structure will have a drastic increase in net profits. This happens because the fixed costs cause total costs to increase at a much slower rate for increasing volumes, resulting in a higher rate of increase for net profits. The break-even can be established graphically or arithmetically.

The Graphical Method

The graphical method could be illustrated as follows:



Mathematical Method

In order to calculate the break-even point of a business mathematically, it is necessary to determine the gross income % of the business or marginal income per unit.

Example:

	TOTAL	PER UNIT
Sales (100 000 units)	R200 000	R2.00
Less variable cost	R120 000	R1.20
Marginal income	R80 000	R0.80
Less: Fixed cost	R20 000	R0.20
Net income	R60 000	R0.60

The marginal income ratio is calculated as follows:

Marginal income ratio = marginal income ÷ income

R 80 000 ÷ 200 000 = 0.4 or 40%

The ratio can now be used to calculate the break-even point.

Break-even point in units = Tot fixed cost ÷ marginal income per unit

= R 20 000 ÷ R 0.80 = 25 000 units

Break-even point in rand = Tot fixed cost ÷ marginal income

= R 20 000 ÷ 40% = R 50 000.00

Learning Unit 4

Forecasting

Unit Standard	
116428	**Analyse and interpret the financial statements and physical records in an agri-business to generate managerial information**
Specific Outcomes	
SO3: Compare financial and economic criteria with historical results and deduct the necessary managerial information	
Learning Outcomes	
• Identify the types and formats of financial forecasts with examples. • Identify sources of financial forecasts • Outline factors in preparing forecasts. • Incorporate relevant factors in the preparation of forecasts. • Analyse financial forecasts to determine viability.	
Critical Cross-field Outcomes	
Organise Collecting Science	Demonstrating Contributing Identifying

INTRODUCTION

Forecasting in most businesses almost always comes down to first forecasting sales volumes (quantity rather than value). Once the sales volumes have been determined, it follows that everything else can be estimated based on this forecast.

For example, if I can forecast how many goods I am going to sell, I can in turn estimate the costs and resources involved in ensuring that we can obtain or produce these goods in the first place. So essentially, forecasts generally revolve around sales, while the rest of the projections made are in actual fact estimations based on these forecasts.

The initial forecast of sales requires educated guesswork, while the estimations based on these forecasts requires arithmetic and logic.

The item that is being forecast may not always be referred to as sales. If the item being forecast is not directly a sales item, it will usually still be an indicator of demand. For example, a non-profit organisation might forecast donations and a government organisation might forecast number of applications.

FORECASTING

Forecasts are often simply done based on past patterns and hunches; however good forecasting utilises a combination of the following two methods:-

1. History of the numbers

   AND

1. Knowledge of the environment (the economy, competitors, the product, your market, etc).

Be careful as sales forecasts often become sales targets and this could in turn impact on the organisation's capacity.

History

When looking at the history of the numbers, you should firstly look for patterns and then project patterns. Patterns could include:-

- The company is expanding.
- The company is in decline.
- The company is seasonal.
- Sales trends.
- Sales cycles.

When projecting patterns, it is important to take note of all the factors that could impact on the existing pattern. For example, an organisation in an expanding pattern might not necessarily continue to expand as it could be reaching its peak and be on the verge of decline. Another example would be a company in decline that has just stepped up its marketing budget and is now on the verge of expansion. Some organisations' patterns might remain the same.

Knowledge of the environment

Once you have determined the existing patterns, you will need knowledge of the market and environment in which you operate in order to project future patterns.

Knowledge of the market and the environment can be obtained from the following:-

- Published economic forecasts and other published forecasts
- Your own research and knowledge
- The media
- Knowledge in your organisation
- The internet
- Industry networks

Some economic indicators you might want also to want to consider when dowing forecasting include:-

- Interest rates: **Interest** is the cost of money. When money is borrowed, the cost involved in using the money is that the borrower will pay back more than the amount that was borrowed. This is true for a personal loan as it is for an investor who borrows capital. Interest is changed for all kinds of 'borrowing' including loans, hire purchase, rental, lease agreements, credit cards, bank overdrafts, and even in case of late payments of accounts
- Exchange rates: Is how much of one currency (Rands) can be exchanged for another currency (US $). If the Rand is poor against the dollar, it is good for exports, but bad for imports. If the Rand is strong against the $ it is bad for exports but good for imports.
- Inflation: Inflation is a sustained and significant increase in general price level. This means that the same R 1.00 which bought you a bread in 1990 can now only buy 1/5 of a bread.
- GDP(Gross Domestic Product) is the total value of goods and services produced by the factors of production located in SA over a specified period. Because some part of domestic production is destined for foreign markets and therefore not sold in South Africa, exports are added to calculate the GDP.
- Prices of commodities
- Unemployment levels
- Consumer spending

A review of your organisations strategies and plans is also important when compiling forecasts.

If your organisation is involved with a large number of items that make up its 'sales', then it is important to group or segment the items first in order to avoid overly detailed forecasts. Items or products that would behave similarly under a given set of conditions can be grouped together. For example, a hardware outlet might forecast sales of hand-held tools and power tools as opposed to forecasting sales for each type of hammer and drill.

Spread sheets can be a helpful forecasting tool.

Individual Formative Exercise 4A

Complete Exercise 4A in your Portfolio of Evidence

ESTIMATING

As discussed previously, once you have forecasted the main element of sales, everything else from your projections follows on from this. The following items form an important part of your projections:

1. Capital spending
2. Employee costs
3. Non-employee costs
4. Other income and costs

Projecting capital spending

Projecting capital spending involves determining what property, vehicles or equipment will be needed to support the sales forecasts.

The following steps should be followed when projecting capital spend:

1. Ask yourself what assets you are going to need.
2. Estimate their expected cost as well as any payment terms.
3. Estimate their useful life spans, residual values and any other costs or benefits.
4. Determine the method of depreciation that you should use.
5. Draw up a depreciation schedule.
6. Set the cost of the acquisitions against your bank balance (offset by an increase in fixed assets).
7. Record depreciation as an expense.
8. Record any other possible expenses, such as maintenance costs or insurance.

9. Take note of any benefits that the assets would have that might impact on other aspects of your projections (for example, better machinery might equate to higher levels of production).

Before we go any further, let us look at the two calculations you need when you project capital spending, i.e. interest calculating and depreciation:

Interest calculation is explained in detail in Learning Unit 6

Depreciation

Depreciation is essentially writing off the costs over the working life of your assets.

Different terms are used to describe writing off costs. 'Depreciation' is used for physical assets, 'Amortization' is used for intangible assets and 'Depletion' is used for natural resources. They are all essentially the same thing.

Methods of depreciation include:-

1. The Straight-Line method

An equal amount of depreciation for each year over the asset's life.

Example:

An R10 000 asset with a five-year life span, no maintenance costs and no residual value.

Divide the depreciable base (the acquisition price plus maintenance costs less residual value) by the number of years of expected life span:

10 000 / 5 = 2 000 per year.

2. Sum-of-the-year's digits.

- -Estimate the number of years of the asset's useful life in reverse order (5,4,3,2,1)
- -Add the reverse order years (5+4+3+2+1 =15)
- Divide each year by the sum (5/15; 4/15; 3/15, 2/15; 1/15)

- Multiply the depreciable base (acquisition cost plus maintenance costs less residual value) by each fraction above (for example, if the depreciable base is R10 000, then 10 000 x 5/15 = 3 333 for the first year and so on.)

3. Double-declining balance

- Calculate the straight-line depreciation
- Calculate the percentage depreciation in the first year and then double it. Using the same example, this would be 2000/10000 x 2 = 0.4 or 40%)
- Use this factor to calculate depreciation on the outstanding balance.
- Use the straight-line figure if the depreciation is higher when calculated using the straight-line method.

The following table illustrates the different depreciation methods:

Straight line method			***Sum-of-years method***				***Double declining balance method***		
Year	**Depreciation**	Net asset value	Year	Factor	**Depreciation**	Net Asset Value	DDB calculation	**Depreciation**	Net asset value
0		10,000				10,000			10,000
1	**2,000**	8,000	5	5/15	**3,333**	6,667	4,000	**4,000**	6,000
2	**2,000**	6,000	4	4/15	**2,667**	4,000	2,400	**2,400**	3,600
3	**2,000**	4,000	3	3/15	**2,000**	2,000	1,400	**2,000**	1,600
4	**2,000**	2,000	2	2/15	**1,333**	667	864	**1,600**	0
5	**2,000**	0	1	1/15	**667**	0	518	**0**	0
Totals	**10,000**		15		**10,000**		9,222	**10,000**	

Source: Stutley, R. 2007. The Definitive Guide to Business Finance. 2nd Edition. Prentice Hall. Pg 168

Use the method of depreciation that is used by your organisation.

Individual Formative Exercise 4B

Complete Exercise 4B in your Portfolio of Evidence

Projecting Employee Costs

Projecting employee costs is fairly simple – use each person's cost to company and take into account any additional employees that may be required based on your forecasts and their costs to company.

Remember that employee costs include the following:-

- Salaries and wage
- Overtime
- Bonuses
- Benefits such as pension and transport allowances
- Other costs such as recruitment costs and training costs.

This information may be readily available from your Human Resources department or you may need to calculate your own breakdowns per staff member.

Projecting non-employee costs

Non-employee costs would include all those costs not associated with employees. Once you have your forecasts, you can estimate these costs.

Examples of operating costs would be:-

- Marketing and sales related costs
- Communications costs such as telephone and internet
- Fees such as legal or accounting fees.
- Occupancy costs such as security, water and electricity.
- Computer costs
- Office costs such as stationery
- Travel costs such as petrol and vehicle rentals.
- Other fees and costs such as insurance and bank charges.

To assist with financial statement projections, you should do separate workings for those employee and non-employee costs that relate to cost of sales and those workings for employee and non-employee costs that relate to other operating costs.

Projecting other Income and Costs

Other income and costs that can be included in your projections include things like investment income and taxation.

Individual Formative Exercise 4C

Complete Exercise 4C in your Portfolio of Evidence

PROJECTED FINANCIAL STATEMENTS

Once you have your forecasts and estimations, you can actually create projections that take the format of your financial statements. Let us look at each one in turn:-

Projected Income Statement

An example of a projected Income Statement is shown below:

Sunnyside Incorporated

Income statement for first six months

Key		Month one	Month two	Month three	Month four	Month five	Month six	Total
A	Sales	-	20 000	-	40 000	-	50 000	**110 000**
B	*Less: Cost of Sales*	-	13 591	-	27 183	-	33 979	**74 753**
C	**Gross**	**-**	**6 409**	**-**	**12 187**	-	**16 021**	**35247**

	Profit							
D	*Less:* **Operating Costs**							
E	Employee costs	54 278	55 400	55 400	62 045	62 045	64 715	**353 883**
F	Other expenditure	12 500	33 814	25 255	17 268	16 596	18 796	**124 229**
G	**Total operating costs**	**66 778**	**89 214**	**80 655**	**79 313**	**78 641**	**83 511**	**478 112**
H	**Net profit (loss) before interest and tax**	**(66 778)**	**(82 805)**	**(80 655)**	**(79 313)**	**(78 641)**	**(67 490)**	**(442 865)**
I	Interest and/ or tax	6 678	8 921	8 066	7 931	7 864	8 351	47 811
J	**Net profit (loss)**	**(73 456)**	**(91 726)**	**(88 721)**	**(74 427)**	**(86 505)**	**(75 841)**	**(490 676)**

Source: Adapted from Stutley R. 2007. The Definitive Guide to Business Finance. 2nd Edition. Prentice Hall. Pg 219

Key	
A	Sales are your sales forecasts as discussed in sections 4.2.1 and 4.2.2
B	Cost of sales needs to be calculated. (For example) Opening stock + Purchases – Closing stock. Cost of sales is calculated based on your sales forecasts.

C	Gross profit projections are your Sales Forecasts less your related Cost of Sales
D	Operating costs include employee and non-employee costs as discussed in sections 4.3.2 and 4.3.3
E	Employee costs would be calculated by using total cost per employee as per current headcount, as well as adding total costs for any additional employees that you project you will need based on your sales forecasts.
F	Other expenditure would include all non-employee costs. You would need to list and add all these to get a total figure. Your list would include things like marketing expenses, travel expenses, depreciation, communication expenses etc. You could list each item separately on your projections or have one total figure as in this example. All of these costs are estimates based on your sales forecasts.
G	Total operating costs is the total of employee and non-employee costs.
H	Net profit or loss before interest and tax is Gross Profit less Total Operating Costs
I	Interest and tax would be calculated based on forecasts and knowledge of the amount of interest earned / owed and tax that needs to be paid. In this example it is shown in one row for the sake of simplicity, however they interest, and tax would be on separate lines.
J	The Net Profit or Loss projections can now be calculated.

Projected Balance Sheet

An example of a simple projected Balance Sheet is shown below:

Assets	**Start**	**Jan**	**Feb**	**March**	**Apr**	**May**	**Jun**
RANDS							
Current Assets							
Cash Balance	55	15	21	7	15	93	53
Accounts Receivable	395	371	474	576	644	803	791
Inventory	251	332	444	545	701	878	647

Other Current Assets	25	25	25	25	25	25	25
Subtotal	***726***	***743***	***964***	***1153***	***1385***	***1799***	***1516***
Capital Assets	350	375	375	390	390	440	440
Accumulated Depreciation	50	51	52	53	54	55	56
Subtotal	300	324	323	337	336	385	384
TOTAL ASSETS	**1026**	**1067**	**1287**	**1490**	**1721**	**2184**	**1900**
Liabilities	**Start**	**Jan**	**Feb**	**March**	**Apr**	**May**	**Jun**
Current liabilities							
Accounts payable	224	268	371	431	564	704	517
Current Notes	90	90	190	220	320	320	220
Other Current Liabilities	15	15	15	15	15	15	15
Subtotal	***329***	***373***	***576***	***666***	***899***	***1039***	***752***
Non-current liabilities	285	282	279	376	373	370	367
Total Liabilities	***614***	***615***	***855***	***1012***	***1272***	***1409***	***1119***
Capital	**Start**	**Jan**	**Feb**	**March**	**Apr**	**May**	**Jun**
Paid-in Capital	500	500	525	525	525	825	825
Retained Earnings	(163)	(88)	(88)	(88)	(88)	(88)	(88)

Earnings	75	0	5	11	12	38	44
Total Capital	412	412	432	448	449	775	781
CAPITAL AND LIABILITIES	**1026**	**1067**	**1287**	**1490**	**1721**	**2184**	**1900**

Source: Adapted from Balance Sheet Example. www.bplans.com/common/gifs/bplans/ IL15-7. Accessed on 19/08/09

Projecting a Balance Sheet requires a little bit more insight than the projection of the Income Statement. The following are some important points to remember for a Balance Sheet Projection:

1. Fixed assets in the Balance Sheet projections forms part of projecting your capital spending as detailed in section 4.3.1.

2. Inventory projections will stem from the sales forecasts, as well as some analysis of production or delivery schedules in relation to your forecasted sales. Inventory projections should also take into account buffer stocks in case there are unexpected changes in demand.

3. The retained earnings entries will change by the amount of net profit in any one period.

4. Accounts payable can be projected as follows:-

2. Accounts payable can be projected as ratios. For example, if you receive an average of the same deliveries each month and your suppliers give you credit of 30 days and you make one payment per month, projected payables will average one-half of one month's spending on supplies.

3. Accounts payable is not a change figure, but an actual figure.

5. Accounts receivable can be projected in a similar manner to accounts payable. Accounts receivable is also not a change figure, but an actual figure.

6. Projecting cash, loans and share capital will depend on your cash flow and financing policies. The following section will cover information on financing policies and Section 4.4.3 will give you some more information on cash flows.

Remember that your two halves of the balance sheet in your projections must balance. If they do not balance you need to revisit your figures.

FINANCING POLICIES

Let us take a moment now to look at some financing policies which will assist you to compile your balance sheet projections.

An organisation will be financed using either debt or equity. The finance may be short-term, medium-term or long term. When projecting financial requirements, you would need to relate back to your sales forecasts and cost estimates in order to determine whether or not you need to finance your operation and how you should finance your operation.

Let us look at some examples of sources of each type of finance:

Short-term finance:

- *Trade credit (buying "on account")*
- *Bank overdraft or bank loans: banks will usually require some form of security for this.*

Medium-term finance:

- *Leasing: monthly payments for purchase or rent of plant, property and equipment.*
- *Hire purchase: Similar to leasing, done through Finance Company.*
- *Medium term-loans: will usually also incur interest and security will need to be provided.*

Long-term finance

- *Long-term loans: will probably be needed to be secured by land or property and will incur interest.*
- *Corporate bonds: Borrowing by a company which is a "paper" traded on secondary financial markets at a fixed interest rate. The borrowing may be secured against specific assets. A debenture is essentially an unsecured bond.*
- *Equity: Shares issued by a company. (Shareholders are owners or part owners of the company)*

An organisation needs to choose how much financing will come from equity and how much financing will come from debt. The following table gives a comparison between the two:

Equity	***Debt***
Investors accept higher risk	Lenders do not like risk
Some ownership and control will need to be given up	Ownership and control is not lost
Dividend payments are optional and are a distribution of equity	Interest has to be repaid and is an expense
Equity can end up costing more than debt, equity investors are looking for a return on equity that is greater than the rate of interest they could earn on a lower-risk investment	

A ratio that can be used to analyse debt is called the *gearing ratio*:

Gearing ratio = Debt

Equity

How to interpret this ratio:

This ratio indicates the extent to which an organisation is dependent on debt or equity. A low gearing indicates a low reliance on debt; a high gearing indicates a high reliance on debt and a possible cash flow problem.

Individual Formative Exercise 4D

Complete Exercise 4D in your Portfolio of Evidence

PROJECTED CASH FLOW STATEMENT

Cash Flow Statements

Introduction

In the previous chapters on Companies we have said that the Cash Flow Statement is part of their financial statements.

The Cash Flow Statement identifies the inflows and outflows of cash during a specific period. The Cash Flow Statement presents the users of financial statements with useful and relevant information, for example, answers to the following

- Was sufficient cash generated from operations to pay the interest charges, dividends and taxation
- How was the expansion financed, etc.

Before we discuss the format of the Cash Flow Statement, we are going to discuss the meaning of certain concepts that are used in the Cash Flow Statement.

Specific Outcomes

At the end of this section you should be able:

1. To define and explain the following concepts:
    - Cash
    - Funds
    - Investment activities
    - Financing activities
    - Operating activities
    - Cash flows from operating activities
    - Cash flows from investing activities
    - Cash flow from financing activities
2. To calculate the following:
    - Cash retained from operating activities
    - Cash utilised in investment activities
    - Cash effects of financing activities
3. To prepare a cash flow statement which fulfil the requirements of statement ACT118.

Concepts

1. **Cash**

   Cash, for the purpose of this statement, is cash at bank and on hand and any other highly liquid investment that are **readily retainable** to known amounts of cash.

2. **Funds**

   Funds can be defined as the financial resources possessed by a company and which now from transactions concluded **third parties.** This means that there can only be a flow of funds if a transaction occurs between the business and a person outside the business. Your main task will be to determine whether there was an inflow or outflow of funds during the past financial year. One can say that the concepts cash and funds go hand in hand.

3. **Investment Activities**

   Investment activities are those activities relating to the acquisition and disposal of fixed assets and investments. You are referred to the increase of R50 000 IN Land and Buildings and the decrease of R10 000 in vehicles in the previous example.

   What will an increase in investments be?

EXAMPLE

	2008	2007
Investments	20 000	10 000

The increase of R 10 000 in investments means that more money was invested in a business outside our business. This means an outflow of funds. The opposite, namely a decrease in investments, will be an inflow of funds.

4. Financing activities

Financing activities are those activities which result in changes in the size and composition Of the debt and capital funding of the business. In other words, the financing activities indicate where the funds were obtained from to finance the investment activities and daily operating activities. The increase in R 20 000 in the Ordinary share capital is a good example of financing activity (example to explain the concept funds).

Will an increase in long term liabilities be an inflow or outflow of funds?

Yes, it will be an inflow of funds because more money is borrowed from outside the business.

5. Operating activities

Operating activities include all transactions and other events that are not investing and financing activities. Cash flows from operating activities are generally the cash effects of transactions and other events that enter into the determination of income.

Examples of these type of activities are:

Dividends paid for the year

Taxation paid for the year and

Interest paid (finance cost) for the year

In our discussion later you will see that the information for operating activities is found in the Income Statement.

6. Cash flows from operating activities

The cash flows from operating activities represent the first section of the Cash flow Statement. In this section all those items which have an effect on the determination of income will be taken into account. The following items are involved.

- Investment income for example dividends received
- Financing charges for example interest paid
- Taxation paid
- Dividends paid
- Changes in working capital which consist of:

- Increase/decrease in inventory
- Increase/decrease in debtors
- Increase/decrease in creditors

- Profit before taxation adjusted with all the non-cash flow items

What is the meaning of the concept non-cash flow items?

Non-Cash Flow items are all those items that have an influence on the profit but that are not a flow of funds because no third party is involved.

The following are examples of non-cash flow items:

- Profit or Loss on disposal of a fixed asset
- Depreciation
- Increase or decrease in the provision for bad debts
- Transfer to reserves

Did you recognise that the majority of the information for this section can be found in the Income Statement? It is only the changes in working capital that are found in the Balance Sheet. Although these changes did not influence the profits directly it must be taken into account in this section because the changes are part of the operating activities of the business.

7. Cash flows from investment activities

The cash flows from investment activities represent the second section of the Cash Flow Statement.

In this section all those items which have an effect on the acquisition and disposal of fixed assets and investments will be taken into account. In other words, this section indicates how the funds were utilised to maintain or to expand operations

The following items are involved:

- Replacement of fixed assets
- Proceeds on the disposal of fixed assets
- Additions to fixed assets
- Investments purchases
- Investments sold

Did you recognise that the information for this section can be in the notes for plant, machinery and equipment (Fixed assets) and Investments? The comparative figures of the previous year must be taken into account to determine the inflow or outflow of funds. (See our discussion later).

8. Cash flows from financing activities

The cash flows from financing activities represent the third section of the Cash Flow Statement. In this section all those items which have an effect on the debt and capital funding of the business, will be taken into account. In other words, this section indicates where the funds were obtained from to Finance the investment activities and the daily operating activities.

The following items are involved:

- Increase or decrease in long term borrowings
- Proceeds from issue of share capital
- Increase in share premium

Did you recognise that the information for this section can be found in the equity and liabilities section of the Balance Sheet? You must determine whether the increase or decreases from the previous year's figures are an inflow or outflow of funds.

Calculations of certain concepts

Before we discuss the format of the Cash Flow Statement according to AC 118 we are first going to discuss how to calculate the following concepts:

- Cash flows from operating activities
- Cash flows from investment activities
- Cash flows from financing activities

To explain the calculations of these concepts only one example will be used.

FORMAT CASH FLOW STATEMENT AC 118

The Cash Flow Statement of a Company must be prepared according to the requirements of Statement AC 118 of the Public Accountants and Auditors Board. As we have indicated before there are two methods namely the indirect and direct method. In this chapter we are going to concentrate on the direct method only as this is the suggested format to use in practice.

LET US DEMONSTRATE

The following information was obtained from the financial statements of Mamoek Limited at 31 March 2008.

INCOME STATEMENT FOR THE YEAR ENDED 31 MARCH 2008

	R '000
Gross turnover	96 000
Cost of Sales	(62 400)
Gross Profit	33 600
Other operating income	1 300
Profit on disposal of Plant	1 300
Investment income	389
Dividends received	164
Interest received	225
Operating costs	(20 089)
Depreciation	12 295
Auditor's remuneration	800
Director's remuneration	1 200
Administrative expenses	5 794
Profit on ordinary activities before interest	15 200
Interest paid	(3 200)
Profit on ordinary activities before taxation	12 000
Taxation	(5 400)
Profit on ordinary activities after taxation	6 600

BALANCE SHEET AT 31 MARCH 2008

	2008	2007
	R'000	R'000
ASSETS		
Non-current assets	126 800	116 000
Fixed assets	121 800	114 500
Investments in shares at cost	5 000	1 500
Current assets	44 680	46 400
Inventory	19 300	21 850
Debtors	25 380	22 750
Bank		1 800
	171 480	162 400
EQUITY AND LIABILITIES		
Capital and Reserves	97 380	81 810
Ordinary share capital	33 000	26 000
Preference share capital	6 000	4 000
Share premium	20 000	18 000
Non-distributable reserves	10 000	10 000
Distributable reserves	28 380	23 180
General reserve	22 800	20 000
Retained income	5 580	3 180
Non-current liabilities	41 280	47 120
Long term loan	21 000	29 000
Debentures	20 280	18 120
Current liabilities	32 820	34 100
Creditors	25 870	27 500

Receiver of Revenue	5 400	6 000
Shareholders for dividends	300	600
Bank overdraft	1 250	
	171 480	162 400

ADDITIONAL INFORMATION

1. FIXED ASSETS

2008 (R'000)

	Land and Building	Plant	Equipment	Total
Gross carrying value	17 100	118 500	4 330	139 930
Acc Depreciation	1 100	16 700	330	18 130
Opening balance	420	5 270	145	5 835
Depreciation	680	11 430	185	12 295
		101	**4**	**121**
Net carrying value	**16 000**	**800**	**000**	**800**

2007 (R'000)

	Land and Building	Plant	Equipment	Total
Gross carrying value	14 420	103 770	3 645	12 335
Acc Depreciation	420	6 770	145	5 835
Opening balance	140	3 000	45	1 685
Depreciation	280	3 770	100	4 150
		97		**114**
Net carrying value	**14 000**	**000**	**3 500**	**500**

2. Plant with a cost price of R2 080 (R'000) and a net carrying value of R580 (R'000) was sold for R1 880 (R'000). Plant purchased was in replacement of the plant sold.

REQUIRED

Prepare the Cash Flow Statement of Mamoek LTD for the year ended 31 March 2008 using both methods (indirect and direct).

VERY IMPORTANT

Did you notice that the information given in Mamoek (LTD) is exactly the same as the information given in SAICA (LTD)? This means that we have already calculated the cash flows from operating activities, from investment activities and from financing activities. All that remain now is to put them together in the correct format according to the requirements of Statement AC 118.

MAMOEK LTD

CASH FLOW STATEMENT FOR THE YEAR ENDED

31 MARCH 2008

	Note	R'000
Cash flows from operating activities		13 585
Cash receipts from customers		93 370
Cash paid to suppliers and employees		(69 274)
Cash generated from operations	1	24 096
Interest received		225
Dividends received		164
Interest paid		(3 200)
Taxation paid (6 000+5 400-5 400)		(6 000)
Dividends paid (600+1 400-300)		(1 700)
Cash flows from investment activities		(21 795)
Purchase of fixed assets/tangible assets		(20 175)
Replacements	2	(16 810)
Additions	3	(3 365)
Proceeds on disposal of fixed assets/tangible assets	4	1 880
Investments purchased		(3 500)
Cash flows from financing activities		5 160

Decrease in long term loans		(8 000)
Proceeds from shares issued	5	11 000
Proceeds from Debentures issued		2 160
Net decrease in cash and cash equivalents		(3 050)
Cash and Cash equivalents at beginning of period		1 800
Cash and cash equivalents at end of period		(1 250)

NOTES TO THE CASH FLOW STATEMENT

1. Reconciliation of net profit before taxation to cash generated from operations.

	R'000
Profit before taxation	12 000
Adjusted for	
Depreciation	12 295
Interest paid	3 200
Profit on disposal of plant	(1 300)
Investment income	(38 9)
Operating profit before changes in working capital	25 806
Changes in working capital	(1 710)
Decrease in inventory	2 550
Increase in debtors	(2 630)
Decrease in creditors	(1 630)
	24 096

2. REPLACEMENT OF FIXED ASSETS/TANGIBLE ASSETS

Plant purchased	16 810

3. ADDITIONS TO FIXED ASSETS/TANGIBLE ASSETS

Land and Buildings purchased	2 680
Equipment purchased	685
	3 365

4. PROCEEDS ON DISPOSAL OF FIXED ASSETS/TANGIBLE ASSETS

Carrying value of asset sold	580
Profit on disposal	1 300
Total proceeds	1 880

5. PROCEEDS FROM SHARES ISSUED

Ordinary shares	7 000
Share premium on ordinary shares	2 000
Preference shares	2 000
	11 000

EXPLANATION

1. You are referred to the explanations of the calculations for SAICA LTD
2. The cash received from customers and cash payments to suppliers and employees are calculated as follows:

Cash received from customers

Debtors

Balance	22 750	*Bank	93 370
Sales	96 000	balance	25 380
	118 750		118 750

All the sales are taken into account as credit sales and the bank is the balancing figure.

Cash payments to suppliers and employees

Creditors

*Bank	61 480	Balance	27 500
Balance	25 870	Purchases	59 850
	87 350		87 350

- Purchases are determined as follows:

Cost of Sales	62 400
Less: Decrease in inventory	(2 550)
	59 850

- The bank is the balancing figure, but this is only the payment to suppliers. What about the other payments to employees? Therefore, the payment to suppliers and employees are calculated as follows:

Payments to suppliers	R61 480
Payments to employees:	
Auditors' remuneration	800
Directors' remuneration	1 200
Admin expenses	5 794
	69 274

Individual Formative Exercise 4E

Complete Exercise 4E in your Portfolio of Evidence

Refer now to the attached example of projections for ABC Limited to see an example of financial forecasting.

Read through this example in order to get a better feel for projections.

The projections that we have discussed here will give you the basic underlying understanding behind how projections are calculated. Projections that you do at the workplace may not be as simple as the one's you have compiled in class. Your organisation might have existing spreadsheets or software in place that does a lot of the workings for you. It is important for you to find out what your organisation has in place with regards to doing projections and to learn how to use your organisation's systems.

Learning Unit 5

Budgeting

Unit Standard	
116428	**Analyse and interpret the financial statements and physical records in an agri-business to generate managerial information**

Specific Outcomes

SO4: Set objectives for different ratios and do an interpretation of the different ratios

Learning Outcomes

- Establish operational objectives in line with the unit's strategic plan.
- Link budget plans to operational objectives.
- Draft budgets according to operational plan of the unit.
- Formulated budget according to standard operating procedures.
- Review and reflect on draft budget and modify to ensure alignment to the operational plan of the unit.
- Agree and adhere to monitoring systems according to standard operating procedures.
- Monitor expenditure reports for the year for each team within a unit against given criteria.
- Implement corrective actions where necessary in accordance with the entity's policies and procedures.

Critical Cross-field Outcomes

Organise	Demonstrating
Collecting	Contributing
Science	Identifying

INTRODUCTION

A budget is a short-term financial plan, usually for a twelve-month period. On approval, a budget ultimately becomes a target and tool for management control.

A budget will record the budgeted figures, the actual figures and the variance between the actual figures and the budgeted figures.

Budgets should ideally not be treated as the only way to control finances, but as one of the tools in a range of measurement control.

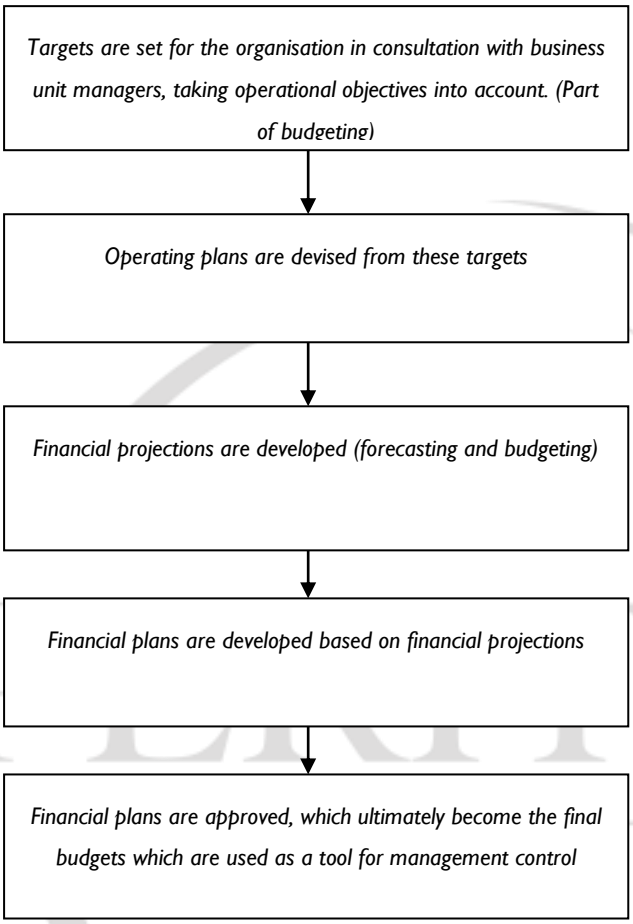
Usually each business unit manager / department manager will prepare a budget. These budgets are then aggregated to form the overall budget for the organisation.

Budgets should be consistent across the organisation, prepared using the same headings expectations and assumptions so that they can be aggregated into the final budget. For example, manufacturing budgets need to be aligned with what sales are budgeting. Also ensure that interest rates or exchange rates are being used consistently across different business units.

WHERE DO BUDGETS FIT IN?

There is quite a bit of overlap between the budgeting process and the forecasting process which was discussed in the previous Learning Unit. The following diagram illustrates, generally speaking, where budgets fit in:

Where do Budgets fit in?



Business unit managers are usually required to report on their progress against their budget each month. This is done by comparing actual spending against budgeted targets. At the end of the budget year, organisations usually conduct more comprehensive comparisons. Often organisations use end of budget year comparisons as performance measurements to reward or "punish" employees, however a better approach would be to incorporate the budget measurement tools with other forms of measurement to reduce the risk of turning budgets into harsh, bureaucratic management tools.

Figures from budgets should be evaluated in conjunction with other organisational elements. These include customer perceptions, strength of processes and ability to innovate. This is sometimes referred to as a "Balanced Scorecard Approach"

THE DEVELOPMENT OF SPENDING PLANS

Each business unit or department will create a spending plan or budget which are ultimately brought together to create the final budget plan for the organisation. Bear in mind the overlap with forecasting which you learnt in the previous Learning Unit.

The process usually takes place as follows:

1. Each business unit draws up their own budgets/ forecasts of expenditure (employee and non-employee costs as discussed in sections 4.3.2 and 4.3.3).
2. Each business unit also draws up their own budgets / forecasts of capital expenditure (discussed in section 4.3.1)
3. In addition to these, the sales business unit will also produce a sales budget / forecast.
4. The production department will also create product volumes and costings budgets/ forecasts.
5. All of these are brought together to form the enterprise budget/ forecast.

If your organisation is small, you may not have all these departments, in which case the final budget can be generated without having to consult with other departments.

Some organisations work with *rolling* budgets as opposed to annual budgets. A rolling budget looks five quarters ahead and is revised every three months. Rolling budgets are often smoother as they take into account re-forecasting that should ideally take place in the course of running a business.

THE MANAGEMENT OF A BUDGET

Managing a budget requires comparing actual figures with budgeted figures (what actually happened compared to what you expected to happen. The difference between the two is called the *variance*. A budget should also include commentary on the variance.

An example of the management of a monthly budget for a business unit is shown below:

Marketing Department - Budget Report June 2008								
Rands								
	Month of June			**Year to end June**				
	Actual	**Budget**	**Variance**	**Actual**	**Budget**	**Variance**		
Salaries	12000	10000	2000	72000	60000	12000		
Stationery	2000	2000	0	12000	12000	0		
Telephone	950	500	450	5700	3000	2700		
Travel	2500	1200	1300	15000	7200	7800		
Total Spending	**17450**	**13700**	**3750**	**104700**	**82200**	**22500**		

Budget Commentary

1. Salaries were over budget by R2000 per month due to the salary increase authorised in line with additional responsibilities allocated.
2. Stationery was line with budget.
3. Telephone expenditure was way over budget by R450 per month – the reasons for this need to be investigated and telephone expenditure needs to be curbed.
4. Travel expenditure exceeded more than 100% of the budgeted figure. Urgently requires a check and control or possibly a revision of budget.

When reviewing a budget, a manager should keep an eye out for some of following problem indicators:-

- Sales are on target, but spending is not.

- Spending is on target, but sales are not.
- Large variances (more than 10%).
- Trends that show a decline, even if still within target.
- Sudden big changes.
- Single items of expenditure that exceed business unit spending limits.
- Large increases in accounts receivable (credit given to customer) and aging accounts receivable (debts not being collected).
- Headings that might conceal payments to third parties.

PLANNING, MONITORING, EVALUATION AND CORRECTIVE ACTION

As a manager, financial management of your unit should be viewed as a management aspect that requires planning, monitoring, evaluation and corrective action.

The old adage "if you can't measure it, you can't manage it" holds true for financial management.

Planning

Budget planning should be done from the bottom up, but frameworks and assumptions for budgeting should be set from the top down. Ensure that individual managers are involved in the budgeting process and are held responsible. It is important to remember that budgets should not be confused with performance management and that budgets should form part of performance management and not the entire yardstick against which people are measured.

Monitoring

An organisation can be affected by a multitude of factors and a manager should be prepared to reset the budget targets as necessary. A manager should look out for trends and measure them (for example, sales are running below target or spending is running above target). Charts can be created to monitor trends.

Non-financial indicators that ultimately affect the numbers should also be monitored, such as headcount, quality control problems and production levels. Ratio analysis also forms an important part of monitoring.

Check numbers ruthlessly and do not assume they are correct just because they have been presented well.

Evaluation

Evaluating requires taking monitoring one step further and is usually done at set intervals. Use the numbers to determine if you are in line with the business strategy. Evaluate whether cost –cutting measures can be put in place and what these measures should be. Ask yourself how efficiency can be improved and if more revenue can be generated. Question where things went wrong and why they went wrong.

A manager should also use their knowledge of financial management to benchmark against competitors.

Corrective Action

Monitoring and evaluation do not mean anything unless corrective action is taken. Managers should be required to prepare reports and action plans based on the budget variances and any ratio analysis they have done.

A typical action plan will include goals, objectives (measurable steps towards achieving those goals), timeframes and persons responsible. An action plan should be monitored and checked to see if items are being actioned on time.

Individual Formative Exercise 5A

Complete Exercise 5A in your Portfolio of Evidence

Learning Unit 6

Financial Instruments

Unit Standards	
7468	**Use mathematics to investigate and monitor the financial aspects of personal business, national and international issues**

Specific Outcomes

SO1: Use mathematics to plan and control financial instruments

SO2: Use simple and Compound interest to make sense of and define a variety of situations

SO3: Investigate various aspects of costs and revenue

Learning Outcomes

SO1:

AC1: Plans are sufficient to ensure effective control of financial instruments

AC2: Calculations are carried out using computational tools effectively and correctly and solutions obtained are verified in terms of context

AC3: Measures used for control purposes are appropriate to the need and are in line with control plans

SO2:

AC1: The differences between simple and compound interest are described in terms of their common applications and effects

AC2: Methods of calculation are appropriate to the problem types

AC3: Computational tools are used efficiently and correctly, and solutions obtained are verified in terms of the context or problem

AC4: Solutions to calculations are used effectively to define the changes over a period of time.

Critical Cross-field Outcomes

• **Organising** • **Collecting**	• **Communicating**

INTRODUCTION:

Organisations use financial instruments to offset potential losses that may be incurred in the future. Different instruments are used for different situations, but all have the purpose to minimise future losses and/or to cover the current situation of a company.

Examples of financial instruments used by companies to minimise risk, include:

- Stocks
- Exchange-traded funds
- Insurance
- Bonds
- Annuities
- Forward Contracts
- Options
- Future Contracts

The term **hedging** refers to the act of reducing a firm's exposure to price or rate fluctuations.

Public future markets were established in the 19th century to allow transparent, standardised and efficient hedging of agricultural commodities such as grains, metals, gas, electricity and oil prices; they have since expanded to include future contracts for hedging the value of energy, precious metals, foreign currency and interest rate fluctuations.

The calculations with regards to hedging instruments are normally done by financial experts in a business or even by actuaries. We will however for the purpose of this unit standard explain the calculations for the forward contract for currencies (simple form of options and futures)

FORWARD CONTRACTS FOR CURRENCIES

The foreign exchange market is a form of exchange for the global decentralised trading of international currencies. Financial centres around the world function as anchors of trading between a wide range of different types if buyers and sellers for currencies. For example, a business in South

Africa may import goods from the United States of America and pay in US Dollars ($), even though its income is in Rand.

Forward Contracts:

One way to deal with the foreign exchange risk is to engage in a forward transaction. In this transaction, money does not actually change hands until some agreed upon future date. A buyer and seller agree on a exchange rate for any date in the future, and the transaction occur on that date, regardless of what the market rates are then. The duration of the transaction can be one day, month or year.

Example:

"A" will receive $10 000 in 3 months' time. The current exchange rate is R8,25 for $1. The market expects that the Rand will weaken over the next 3 months (maybe R7,90). "A" can cover this potential loss in a forward cover deal. "A" and "B Bank" agrees on R8,30 for $1 After the 3-month period on a specific date.

For "B Bank" to offer this rate, he uses an interest financial instrument/investment.

"A" must have controls and plans in place to evaluate the current markets to ensure efficiency of the financial instruments he uses.

INTEREST CALCULATIONS:

In order to analyse the financial information of this unit, we first need to look at the calculations of simple and compound interest.

Interest is the cost of money. When money is borrowed, the cost involved in using the money is that the borrower will pay back more than the amount that was borrowed. This is true for a personal loan as it is for an investor who borrows capital. Interest is changed for all kinds of 'charge' including loans, hire purchase, rental, lease agreements, credit cards, bank overdrafts, and even in case of late payments of accounts.

- The capital on which the interest is calculated at the beginning of the transaction is called the **principle (P)** or present **value (PV)**
- The rate of **interest (r)** is that percentage of the principle that is to be paid for each unit of time and is expressed as a percentage per year.
- The **time period (t)** is the period for which the money is borrowed and is expressed in years or a fraction of a year.
- The amount to be paid at the end of the term, that is, the principle plus the interest, is referred to as the **amount (A)**, or the future **value (FV)**.
- Interest can be calculated on the principle sum as:
    - Simple interest or
    - Compound interest

Simple Interest

When simple interest calculations are done, interest is calculated on the principal sum only at the end of a specified period, such as at the end of a year. That means the interest is not available before the end of the term, and the interest is not added to the principal to earn interest on interest.

The standard formulae for calculating simple interest are:

$I = Prt$

$A = P (1 + rt)$

$A = P + I$

Where:

I = Amount of interest	r = Interest rate per annum expressed as a decimal
P = Principal	t = Time in years or a portion of a year

A = Amount	

Example:

See forward Contract deal in the previous example for "B Bank".

"B Bank" will make an investment with "C Bank" in the amount of R 82 500

(R8,25 x $ 10 000), for 3 months (90 days) at 7% interest per year.

A = P (1 + rt)

A = 82 500 (1 + (0,07 x 90/365)

= 82 500 (1,0173)

= R 83 924

The interest earned over the 3-month period is R 83 924 less R 82 500 = R 1 424.

"B Bank" uses this to pay "A" his R 83 000 (R 8,30 x $ 10 000) and keep the profit. "B Bank"

Then uses the $ 10 000 for further financial deals.

Compound Interest

When interest is not paid out, but is continuously added to the principal, the principal is continuously increasing, and we say the interest is compound. This means that the interest calculated in period 1 on the principal amount is added to the principal amount so that the interest calculated in period 2 is calculated on the increased balance. It can therefore be said that compound interest calculates 'interest in interest'. Compound interest will therefore be more than simple interest, even if the % of interest is the same.

Interest can be compounded once a year, semi-annually, quarterly, monthly or even daily. The time period, which is normally quoted as a yearly rate, should be adjusted to the number of interest periods per transaction. For example, if the interest is compounded quarterly, and the time period is 5 years, then the number of interest-compounding periods

(n) is 5 x 4 = 20.

To obtain the period rate (i) from the yearly rate (r), the average rate per period method is followed: for example, if the annual rate is 6% compounded quarterly, the period rate is taken to be 6/4 = 1.5%.

N = no. of years x no. of compounding periods per year

I = annual rate / no. of periods per year

The standard formulae for calculating compound interest are:

$A = P(1 + i)^n$

$I = (A/P)^{1/n} - 1$

$$T = \frac{\log A/P}{\log (1 + i)}$$

Where:

A = Amount or future value

P = Principal or present value

i = Interest rate per period within a year expressed as a decimal

n = number of times per year interest must be calculated

Note: Doing compound interest calculations will require the use of the 'power' key on the calculator, which will be marked as either x^y or y^x, the 'root' key, which is usually marked as $\sqrt{}$, and the 'log' key.

Example:

Simon lends R 1 000 to Thandi. At a rate of 15% per annum calculated monthly, the amount she must repay at the end of 2 years is as follows:

The interest rate of 15% is the interest that is charged for the year. However, if the interest is to be calculated monthly, then the annual interest rate (15%) must be converted to a monthly interest rate by dividing by 12:

i = 15%/12 = 1,25%

The 2-year period should change to n = 12 x 2 = 24

$A = 1\,000\,(1 + 1{,}25\%)^{24} = 1\,000\,(1{,}0125)^{24} = R\,1\,347.35$

Amount of interest paid: R1 347.35 – R 1 000.00 = R 347.35

Compound interest is used in mortgage loans and hire purchases and we need to repay the loan over the period of time, the following formula will apply:

$$R = P \frac{i}{[1-(1+i)^{-n}]}$$

Example:

Lerato successfully completed her studies and found a job as an IT technician. She wanted to buy a new Chevrolet Aveo. She needed to obtain financing from the bank to the value of R90 000 to buy the vehicle. The monthly compounded interest rate is 12%. She will amortize the loan by monthly payments over a period of 4 years.

$$R = 90\,000 \frac{[0.12/12]}{[1-(1+.12/12)]}$$

$$= 90\,000(0.02633835)$$

$$= R2370.05$$

ANNUITIES

An annuity is a sequence of equal payments made at equal time intervals, such as instalment payments, pensions, insurance premiums, home loan payments, rent, etc. The time between successive payment (R) is called the payment interval, and the time between the first payment and the last payment is called the term of the annuity. The payment interval and the interest period always coincide, which means that, if the interest is compounded monthly, the payments will be monthly.

Annuities are classified into three main classes:

- Ordinary annuities certain refer to annuities where the regular payments are made at the end of each payment interval.
- Ordinary annuities due refer to annuities where the periodic payment (R) falls at the beginning of each payment interval.
- Deferred annuities refer to annuities where the first payment is not made at the end of the first interest period, but at some later date.

Ordinary annuities certain:

The regular payments made are at the end of each payment period. To calculate the future value or amount (A) of an ordinary annuity certain, we apply the following formula:

$$A = R \frac{(1 + i)^n - 1}{i}$$

To calculate the present value or principal (P) of an ordinary annuity certain, we apply the following formula:

$$P = R \frac{1 - (1 + i)^{-n}}{i}$$

Example:

Determine the amount of an annuity certain of R150 per quater for 3 years if the money is worth 12% compounded quarterly:

$$A = R \cdot \frac{(1 + i)^n - 1}{i} = 150 \cdot \frac{(1 + 3\%)^{12} - 1}{3\%} = 2\ 128.80 \text{ Rands}$$

Ordinary annuities due

If the periodic payments fall at the beginning of each payment period (pay in advance), the following formulae apply:

To calculate the amount or future value:

$$A = R \frac{[(1 + i)^n - 1][1 + i]}{i}$$

To calculate the present value or principal:

$$R = \frac{Ai}{[(1+i)^n - 1][1 + i]}$$

Example:

An investment of R 200 is made at the beginning of a year for 10 years. If interest is 12%, how much

will the investment be worth at the end of 10 years?

$$A = R \frac{[(1 + i)^n - 1][1 + i]}{i} = 200 \frac{[(1 + 12\%)^{10} - 1][1 + 12\%]}{12\%} = 3\,930.92 \text{ rand}$$

Individual Formative Exercise 6A: Financial Instruments

Learning Unit 7

The National and Global Economy

Unit Standards	
7468	**Use mathematics to investigate and monitor the financial aspects of personal business, national and international issues**
Specific Outcomes	
SO4: Use mathematics to debate aspects of the national and global economy	
Learning Outcomes	
AC1: Values are calculated correctly AC2: Mathematical tools and systems are used effectively to determine, compare and describe aspects of the national and global economy AC3: Debating points are based on well-reasoned arguments and are supported by mathematical information	
Critical Cross-field Outcomes	
• Organising • Collecting	• Communicating

MONEY VALUE VS VALUE IN REAL TERMS

The main reason for calculating national income is to provide an indication of the actual quantity (or volume) of goods and services produced during a particular year' and compare them to previous years.

Look at the table below:

	Total output Yo-Yo's (Rand)	Index of prices
Year 1	100 000	100
Year 2	120 000	120
Year 3	150 000	132

Between year 1 and year 2 the money value of the total production of yo-yo's increased by 20% from R100 000 to R120 000. The price of yo-yo's also rose by 20% over the same period, so the value of the total output in year 2 in terms of year 1 prices is:

R120 000 x 100/120 = R 100 000

This calculation enables us to remove the effect of inflation in the numbers. This calculation shows us that no increase in production took place between years 1 and 2. In year 3 the money value of the total production of yo-yos rose by another 25% (to R150 000), while prices increased by 10% (132-120/120 x 100). The value of the output in year 3 in terms of year 1 prices is:

R150 000 x 100/132 = R113 636

Although the monetary value of total yo-yo production rose by 50%(nominal terms) between year 3 and 1, the actual volume of production increased by only 13.63%(real terms)

NATIONAL ACCOUNTS

National accounts building blocks are Gross Domestic Expenditure (GDE) and Gross Domestic Product (GDP).

GDE: is the total spending within an economy and there are three types of buyers of goods and services, namely.

Personal Consumption Expenditure (PCE), i.e. Households who purchase consumer goods and utilise services.

Economic agents, i.e. goods used for Capital formation, including Gross Domestic Fixed Investments (GDFI) as well as the value of the change in inventories.

Government Consumption Expenditure(GCE) (largest portion on wages and salaries of civil servants).

GDE = PCE + Investment + GCE + Residual item

(Investment = GDFI and the change in inventories)

(Residual item = adjustment to ensure the expenditure method and income method outcomes are the same).

GDP: is the total value of goods and services produced by the factors of production located in SA over a specified period. Because some part of domestic production is destined for foreign markets and therefore not sold in South Africa, exports are not included in the GDE and must be added to calculate the GDP.

On the other hand, some final expenditure is on goods imported from abroad. But domestic product is the total of goods produced in the country, and imports should be excluded. So, to calculate GDP, the following equation is used:

GDP = GDE + Exports – Imports

To determine the country's economic growth rate for a particular year, the percentage change in the real GDP between that year and the previous year is calculated, as shown in the table below:

Year	Real GDP	Economic Growth (%)
1987	259,6	
1988	270,5	4,2
1989	276,9	2,4
1990	276,1	-0,3
1991	273,2	-1,1
1992	267,3	-2,2
1993	270,2	1,1
1994	276,5	2,3
Ave 1987-1994		0,9

INFLATION

Inflation is a sustained and significant increase in general price level – Andre Roux

This definition implies that only when price increases are recorded for a wide range of goods and services, does inflation occur.

The general price level is measured by the 'Consumer Price Index' (CPI). To calculate the CPI, some 600 goods and services are included in a 'basket'. The 'weight' of each is determined by surveys of household expenditure. The two items that weighs most are food and housing.

The consequences of inflation

The table below shows how inflation has eroded the purchasing power of your money since 1970:

Year	CPI	Purchasing power of 1970's R1000
1970	100	R1 000
1975	157	R 637
1980	277	R 361
1985	533	R 188
1990	1 086	R 92
1994	1 705	R 59

The second column of the table shows that price of a basket of goods costing R 100 in 1970, was R 1705 in 1994. Put differently, the average price level in 1994 was 17 times higher than in 1970.

The third column in the table shows how the purchasing power of R 1 000 has declined over the mentioned two decades. In 1970 R 1 000 enabled you to buy 1 000 goodies, ten years later that same R 1 000 would buy 361 goodies; while in 1994 you could only buy 59 goodies with your hard-earned R 1 000.

In 1970-1994, South Africa's inflation rate was very high, reaching the highest scale of 15% by 1988. The consequences of persistently high inflation are always negative for the economy:

First, and most obvious, is that inflation reduces your purchasing power. If disposable income rises by say 10% every year while inflation averages 15% per year, then your purchasing power actually declines by roughly 5% a year.

Inflation favours debtors at the expense of creditors. Let's assume you entered into a hire-purchase agreement where you have to make 60 monthly payments of R 100 each to a supplier; with the annual inflation rate of 15%. In a year's time the actual purchasing power of your R 100 would have fallen to R 85; and by year 5 your R 100 will be worth R 50. In effect you are paying your debt with cheaper money. The seller is affected in the exact opposite way – the instalments he receives from you is declining because of inflation. In short, inflation redistributes wealth – taking away from creditors in favour of debtors.

Inflation is also detrimental for exchange rate of the rand. If the cost of goods produced in SA rises by 15% per year, while the cost of producing the same goods in the USA rises by 5% per year, then it means that local goods become relatively more expensive. As a result, our revenue could decline because the international demand for our goods will drop as foreigners elect to buy the relatively cheaper American items. By the same token, South African importers will prefer to buy the goods produced in America rather than the relatively more expensive locally produced ones. Now if imports rise and export falls, this will invariable lead to the depreciation of the Rand, which leads to higher inflation – a vicious cycle!

In conclusion: inflation can be a vicious cycle, and everything possible must be done to curb price rises. For consumers, this means avoiding, as far as possible, the use of credit to finance the purchase of goods, especially so-called luxury items.

MONETARY POLICY

Monetary policy is a broad concept covering a number of measures implemented by the South African Reserve Bank to achieve its economic objectives. These objectives are regulating price fluctuations (controlling inflation); the balance of payments; the international value of the rand; and employment.

The idea is that by stabilising the short-term performance of these variables, monetary policy aims to affect the money supply, credit extension by financial institutions to borrowers and interest rates.

One of the decisions the Reserve Bank has to make, which are often debated, is what to do with the interest rate. Some factors that play a role in determining the level of long-term interest rates, include:

- Perceptions about the stability of the South African economy
- Expectations about the inflation
- Short term interest rates

If the economy is perceived to be stable, and if inflation as well as short–term interest rates are expected to fall, long term interest rates tend to decrease.

On the other hand, if the South African economy is seen to be unstable and an increase in inflation and short-term interest rates are expected, long-term interest rates usually rise.

Now you may ask the question: How can a rise in interest rates in the USA cause the rand to depreciate which, in turn, may lead to higher inflation in South Africa and a renewed bout of rand depreciation?

Let's assume that the prime overdraft rate in South Africa is 15% and in the USA 12%. A South African importer wishes to buy goods abroad, but he needs to borrow money to finance his purchases. Assuming that he has access to foreign financing, he would prefer to borrow the money from a bank in the USA – it is cheaper. In this case, foreign funds are used to finance South African imports and there will be no outflow of rands. If the prime rate in the USA rises to 18%, while South African prime rate remains unchanged at 15%, local importers will prefer to borrow money from South African banks. These rands are offered in exchange for dollars(used to buy the goods from sources overseas). This means that rands become cheaper – the rand depreciates. And if the rand depreciates significantly, inflation rates often rises; which can lead to further depreciation of the rand.

So, because the rising of foreign interest rates can lead to the depreciation of the rand and even rise in inflation, you will find that the Reserve Bank may decide to raise local interest rates should the interest rates in the USA, the European union, Japan or England (some of our major trading partners) rise.

Next time you listen to the economic news or read the financial pages of the newspapers, you will have a good idea of what's being said or written!

Individual Formative Exercise 7A: The national and international economy

PART B

Learning Units 8 and 9

Mathematics

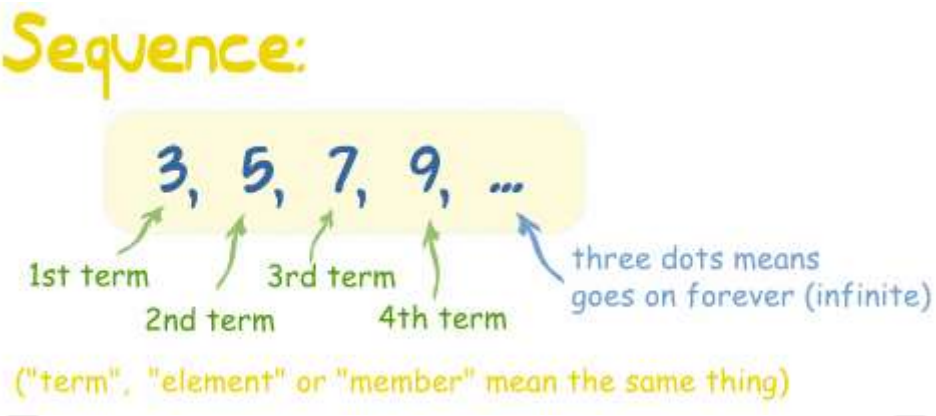
Learning Unit 8

Number sequences and series

Unit Standards	
7483	**Solve problems involving sequences and series in real and simulated situations**
Specific Outcomes	
SO1: Use sequences and series to model real and simulated situations and interpret their findings SO2: Apply models based on sequences and series to solve problems SO3: Investigate and interpret convergence of sequences and infinite geometric series	
Learning Outcomes	
• Understand number sequences and series • Understand finite and infinite sequence and series • Understand the characteristics of a number series • Understand the different types of number sequences	
Critical Cross-field Outcomes	
• Identifying • Collecting	• Communicating

WHAT IS A SEQUENCE?

A Sequence is a set of things (usually numbers) that are in order.



INFINITE OR FINITE

If the sequence goes on forever it is called an **infinite sequence**, otherwise it is a **finite sequence**

Examples:

{1, 2, 3, 4 ,...} is a very simple sequence (and it is an **infinite sequence**)

{20, 25, 30, 35, ...} is also an infinite sequence

{1, 3, 5, 7} is the sequence of the first 4 odd numbers (and is a **finite sequence**)

{4, 3, 2, 1} is 4 to 1 **backwards**

{1, 2, 4, 8, 16, 32, ...} is an infinite sequence where every term double

{a, b, c, d, e} is the sequence of the first 5 letters **alphabetically**

{f, r, e, d} is the sequence of letters in the name "fred"

{0, 1, 0, 1, 0, 1, ...} is the sequence of **alternating** 0s and 1s (yes, they are in order, it is an alternating order in this case)

THE CHARACTERISTICS OF A NUMBER SEQUENCE:

- **In Order**

When we say the terms are "in order", we are free to define **what order that is**! They could go forwards, backwards ... or they could alternate ... or any type of order you want!

- **Like a Set**

A Sequence is like a Set, except:

- the terms are **in order** (with Sets the order does not matter)
- the same value can appear many times (only once in Sets)

Example: {0, 1, 0, 1, 0, 1, ...} is the **sequence** of alternating 0s and 1s.

The **set** would be just {0,1}

- **Notation**

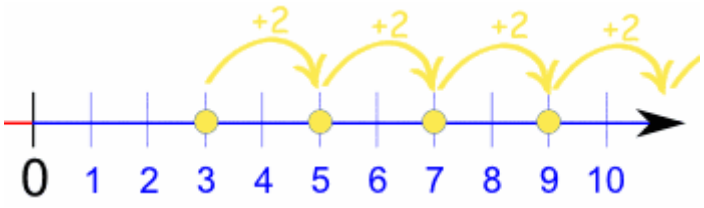
Sequences also use the same **notation** assets: list each element, separated by a comma,
and then put curly brackets around the whole thing.

{3, 5, 7, ...} The curly brackets { } are sometimes called "set brackets" or "braces".

- **A Rule**

A Sequence usually has a **Rule**, which is a way to find the value of each term.

Example: the sequence {3, 5, 7, 9, ...} starts at 3 and jumps 2 every time:



- **Can be expressed as a Formula**

Saying "*starts at 3 and jumps 2 every time*" is fine, but it doesn't help us calculate the:

- 10th term,
- 100th term, or
- ***n***th term, where ***n*** could be any term number we want.

So, we want a formula with "n" in it (where n is any term number).

So, What Would A Rule For {3, 5, 7, 9, ...} Be?

Firstly, we can see the sequence goes up 2 every time, so we can **guess** that a Rule will be something like "2 times n" (where "n" is the term number). Let's test it out:

Test Rule: 2n

n	Term	Test Rule
1	3	2**n** = 2×**1** = **2**
2	5	2**n** = 2×**2** = **4**
3	7	2**n** = 2×**3** = **6**

That **nearly** worked ... but it is **too low** by 1 every time, so let us try changing it to:

Test Rule: 2n+1

n	Term	Test Rule
1	3	2**n**+1 = 2×**1** + 1 = 3
2	5	2**n**+1 = 2×**2** + 1 = 5
3	7	2**n**+1 = 2×**3** + 1 = 7

That Works!

So instead of saying "starts at 3 and jumps 2 every time" we write this:

2n+1

Now we can calculate, for example, the **100th term**:

2 × 100 + 1 = **201**

Many Rules

But mathematics is so powerful we can find **more than one Rule** that works for any sequence.

Example: the sequence {3, 5, 7, 9, ...}

We have just shown a Rule for {3, 5, 7, 9, ...} is: **2n+1**

And so we get: {3, 5, 7, 9, 11, 13, ...}

But can we find another rule?

How about **"odd numbers without a 1 in them"**:

And we would get: {3, 5, 7, 9, 23, 25, ...}

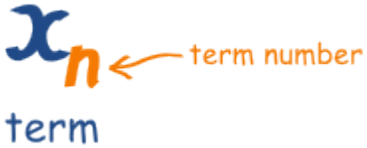
A completely different sequence!

And we could find more rules that match **{3, 5, 7, 9, ...}**. Really, we could.

So it is best to say "A Rule" rather than "The Rule" (unless you know it is the right Rule).

Notation

To make it easier to use rules, we often use this special style:



- $\mathbf{x_n}$ is the term
- **n** is the term number

Example: to mention the "5th term" you just write: x_5

So a rule for **{3, 5, 7, 9, ...}** can be written as an equation like this:

$$x_n = 2n+1$$

And to calculate the 10th term we can write:

$$x_{10} = 2\mathbf{n}+1 = 2\times\mathbf{10}+1 = 21$$

Can you calculate x_{50} (the 50th term) doing this?

Here is another example:

Calculate the first 4 terms of this sequence:

$$\mathbf{\{a_n\} = \{ (-1/n)^n \}}$$

Calculations:

- $a_1 = (-1/1)^1 = -1$
- $a_2 = (-1/2)^2 = 1/4$
- $a_3 = (-1/3)^3 = -1/27$
- $a_4 = (-1/4)^4 = 1/256$

Answer:

$$\{a_n\} = \{ -1, 1/4, -1/27, 1/256, ... \}$$

SPECIAL SEQUENCES

Now let's look at some special sequences, and their rules.

Arithmetic Sequences

In an Arithmetic Sequence **the difference between one term and the next is a constant**.

In other words, you just add some value each time ... on to infinity.

Example:

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number.

Its Rule is $\mathbf{x_n = 3n-2}$

In General, you could write an arithmetic sequence like this:

{a, a+d, a+2d, a+3d, ... }

where:

- **a** is the first term, and
- **d** is the difference between the terms (called the **"common difference"**)

And you can make the rule by:

$$x_n = a + d(n-1)$$

(We use "n-1" because **d** is not used in the 1st term).

Geometric Sequences

In a Geometric Sequence each term is found by **multiplying** the previous term by a **constant**.

Example:

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a factor of 2 between each number.

Its Rule is $\mathbf{x_n = 2^n}$

In General, you could write a geometric sequence like this:

$$\{a, ar, ar^2, ar^3, ... \}$$

where:

- **a** is the first term, and
- **r** is the factor between the terms (called the **"common ratio"**)

Note: **r** should not be 0.

- When **r=0**, you get the sequence {a,0,0,...} which is not geometric

And the rule is:

$$x_n = ar^{(n-1)}$$

(We use "n-1" because ar^0 is the 1st term)

BINOMIALS

A Binomial is a mathematical expression consisting of two terms connected by a plus sign or minus sign.

A **term** is a combination of numbers and variables. In the example $3x + 5$, our first term is $3x$, and our second term is 5. Terms are separated by either **addition** or **subtraction**. In our first example, notice how the $3x$ and 5 are separated by addition. In the last example, we have a binomial whose two terms both have the same variable s. Notice how each term has its variable to a different exponent. The first term has an exponent of 5, and the second term has an exponent of 4. While we can have fractions for our numbers, we cannot have fractional exponents.

All of these examples in the block below are binomials.

$$3x + 5$$
$$x^2 - y^2$$
$$3x + 3y$$
$$t^4 - \frac{7}{9}t^5$$
$$s^5 + \frac{s^4}{3}$$

Examples of binomials.

The following is a list of what binomials must have:

- They must have two terms.
- If the variables are the same, then the exponents must be different.
- Exponents must be whole positive integers. They cannot be negatives or fractions.

Binomial Theorem

What happens when you multiply a binomial by itself ... many times?

Here is the answer:

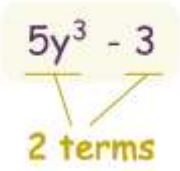
$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Don't worry ... I will explain it all!

And you will learn lots of cool math symbols along the way.

Binomial

A binomial is a polynomial with two terms



example of a binomial

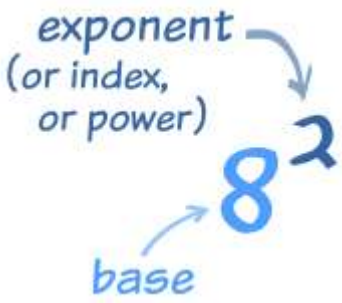
Multiplying

The Binomial Theorem shows what happens when you multiply a binomial by itself (as many times as you want).

It works because there is a pattern ... let us see if we can discover it.

EXPONENTS AND COEFFICIENTS

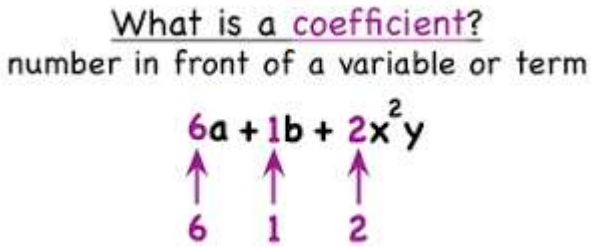
But first you need to know what an Exponent is.
Here is a quick summary:



An exponent says **how many times** to use something in a multiplication.

In this example: **$8^2 = 8 \times 8 = 64$**

A coefficient is a number used to multiply a variable.



Example: 6z means 6 times z, and "z" is a variable, so 6 is a coefficient.

Exponents

An exponent of **1** means just to have it appear once, so you get the original value:

Example: $8^1 = 8$

An exponent of **0** means not to use it at all, and we have only 1:

Example: $8^0 = 1$

Exponents of (a+b)

Now on to the binomial.

We will use the simple binomial a+b, but it could be any binomial.

Let us start with an exponent of **0** and build upwards.

Exponent of 0

When an exponent is 0, you get **1**:

$$(a+b)^0 = 1$$

Exponent of 1

When the exponent is 1, you get the original value, unchanged:

$$(a+b)^1 = a+b$$

Exponent of 2

An exponent of 2 means to multiply by itself:

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

Exponent of 3

For an exponent of 3 just multiply again:

$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3$$

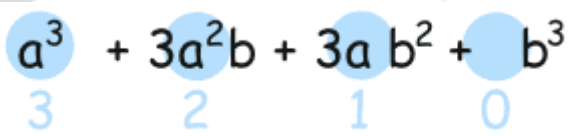
We have enough now to start talking about the pattern.

The Pattern

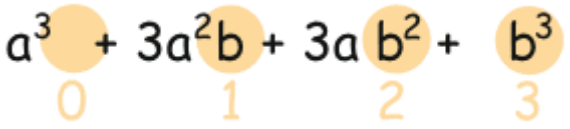
In the last result we got:

$$a^3 + 3a^2b + 3ab^2 + b^3$$

Now, notice the exponents of a. They start at 3 and go down: 3, 2, 1, 0:



Likewise the exponents of b go upwards: 0, 1, 2, 3:



If we number the terms 0 to *n*, we get this:

k=0	k=1	k=2	k=3
a^3	a^2	a	1
1	b	b^2	b^3

Which can be brought together into this:

$$a^{n-k}b^k$$

How about an example to see how it works?

Example: When the exponent, n, is 3.

The terms are:

k=0:	k=1:	k=2:	k=3:
$a^{n-k}b^k$	$a^{n-k}b^k$	$a^{n-k}b^k$	$a^{n-k}b^k$
$= a^{3-0}b^0$	$= a^{3-1}b^1$	$= a^{3-2}b^2$	$= a^{3-3}b^3$
$= \mathbf{a^3}$	$= \mathbf{a^2b}$	$= \mathbf{ab^2}$	$= \mathbf{b^3}$

It works like magic!

Coefficients

So far we have: $a^3 + a^2b + ab^2 + b^3$

But we **really** need: $a^3 + 3a^2b + 3ab^2 + b^3$

We are **missing the numbers** (which are called *coefficients*).

Let's look at **all the results** we got before, from $(a+b)^0$ up to $(a+b)^3$:

$$1$$

$$a + b$$

$$a^2 + 2ab + b^2$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

And now look at **just the coefficients** (with a "1" where a coefficient wasn't shown):

$$1$$

$$1a + 1b$$

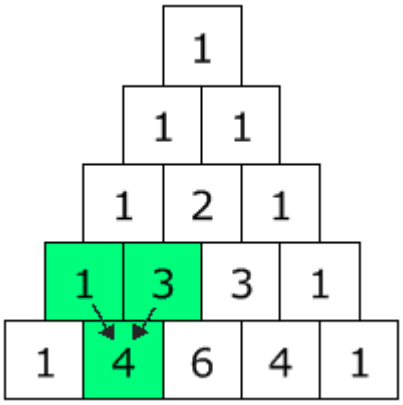
$$1a^2 + 2ab + 1b^2$$

$$1a^3 + 3a^2b + 3ab^2 + 1b^3$$

They actually make Pascal's Triangle!

Each number is just the two numbers above it added together (except for the edges, which are all "1")

(Here I have highlighted that **1+3 = 4)**



Armed with this information let us try something new ... an **exponent of 4**:

a exponent go 4,3,2,1,0: $a^4 + a^3 + a^2 + a + 1$

b exponents go 0,1,2,3,4: $a^4 + a^3b + a^2b^2 + ab^3 + b^4$

coefficients go 1,4,6,4,1: $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ ✓

And that is the correct answer.

We have success!

We can now use that pattern for exponents of 5, 6, 7, ... 50, ... 112, ... you name it!

As a Formula

Our last step is to write it all as a formula.

But hang on, how do we write a formula for **"find the coefficient from Pascal's Triangle"** ... ?

Well, there **is** such a formula:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

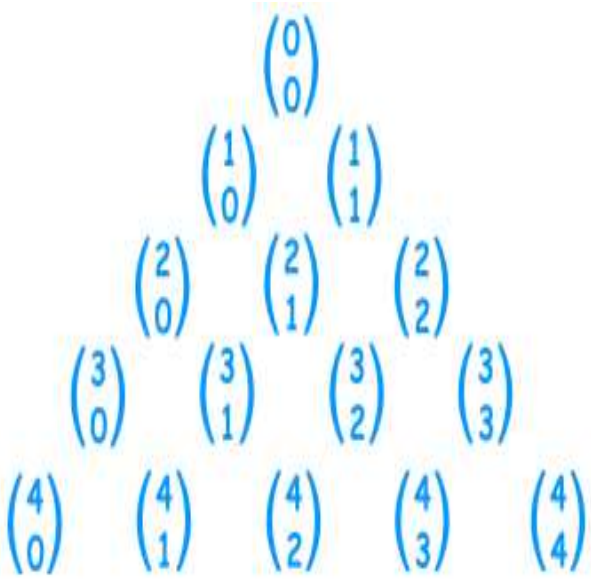
It is commonly called "n choose k" because it is how many ways to choose k elements from a set of n.

You can read more at Combinations and Permutations

The "!" means "factorial", for example 4! = 1×2×3×4 = 24

And it matches to Pascal's Triangle like this:

(Note how the top row is row zero and also the leftmost column is zero!)



Example: Row 4, term 2 in Pascal's Triangle is "6".

Let's see if the formula works:

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$$

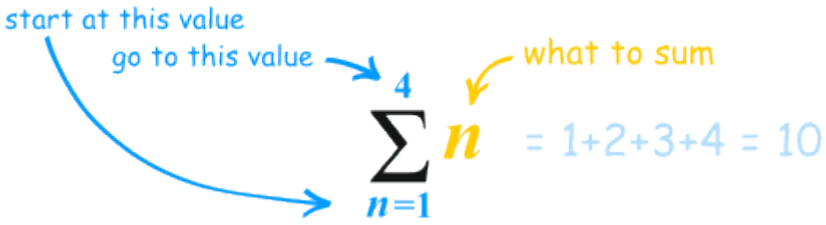
Yes, it works! Try another value for yourself.

Putting It All Together

The last step is to put all the terms together into **one formula**.

But we are adding lots of terms together ... can that be done using one formula?

Yes! The handy Sigma Notation allows us to sum up as many terms as we want:



Sigma Notation

Now it can all go into one formula:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

The Binomial Theorem

Use It

OK ... it won't make much sense without an example.

So, let's try using it for *n* = 3 :

$$\begin{aligned}(a+b)^3 &= \sum_{k=0}^{3} \binom{3}{k} a^{3-k} b^k \\ &= \binom{3}{0} a^{3-0} b^0 + \binom{3}{1} a^{3-1} b^1 + \binom{3}{2} a^{3-2} b^2 + \binom{3}{3} a^{3-3} b^3 \\ &= 1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3 \cdot a^1 b^2 + 1 \cdot a^0 b^3 \\ &= a^3 + 3a^2 b + 3ab^2 + b^3\end{aligned}$$

BUT ... it is usually **much easier** just to remember the **patterns**:

- The first term's exponents start at **n and go down**
- The second term's exponents start at **0 and go up**
- Coefficients are from Pascal's Triangle, or by calculation using **n!/(k!(n-k)!)**

Like this:

Example: What is $(x+5)^4$

Start with exponents:	$\mathbf{x^4 5^0}$	$\mathbf{x^3 5^1}$	$\mathbf{x^2 5^2}$	$\mathbf{x^1 5^3}$	$\mathbf{x^0 5^4}$
Include Coefficients:	$\mathbf{1}x^4 5^0$	$\mathbf{4}x^3 5^1$	$\mathbf{6}x^2 5^2$	$\mathbf{4}x^1 5^3$	$\mathbf{1}x^0 5^4$

Then write down the answer (including all calculations, such as 4×5, 6×5^2, etc):

$$(x+5)^4 = x^4 + 20x^3 + 150x^2 + 500x + 625$$

You may also want to calculate just one term:

Example: What is the coefficient for x^3 in $(2x+4)^8$

The **exponents** for x^3 are:

$$(2x)^3 4^5$$

The **coefficient** is "8 choose 3". We can use Pascal's Triangle, or calculate directly:

$$\frac{\mathbf{n!}}{\mathbf{k!(n-k)!}} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8\times7\times6}{3\times2\times1} = \mathbf{56}$$

And we get:

$$56(2x)^3 4^5$$

Which simplifies to:

$$\mathbf{458752}\ x^3$$

A large coefficient, isn't it?

And one last, most amazing example:

Example: A formula for e (Euler's Number)

You can use the Binomial Theorem to calculate e (Euler's number).

e = **2.718281828459045...** (the digits go on forever without repeating)

It can be calculated using:

$$(1 + 1/n)^n$$

(It gets more accurate the higher the value of **n**)

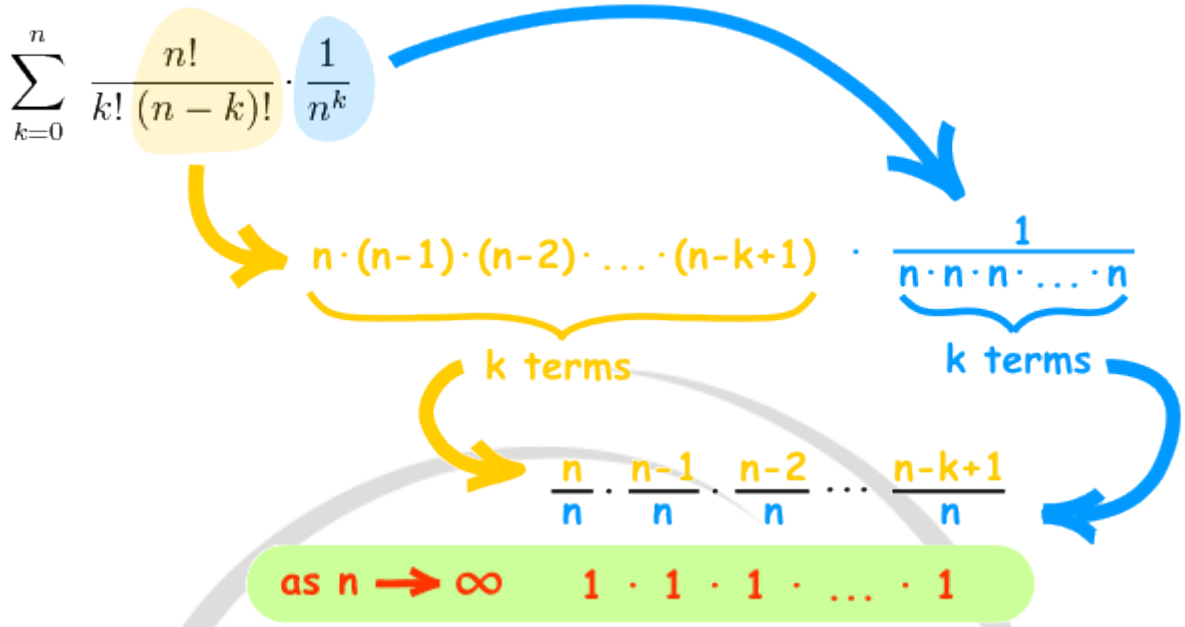
That formula is a **binomial**, right? So let's use the Binomial Theorem:

$$(1+\tfrac{1}{n})^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} (\tfrac{1}{n})^k$$

First, we can drop 1^{n-k} as it is always equal to 1:

$$= \sum_{k=0}^{n} \binom{n}{k} (\tfrac{1}{n})^k$$

And, quite magically, most of what is left goes to **1** as n goes to infinity:



Which just leaves:

$$\sum_{k=0}^{\infty} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$$
$$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \ldots$$

With just those first few terms we get e ≈ 2.7083...

Try calculating more terms for a better approximation!

Individual Formative Exercise 8A: Sequence and series

Learning Unit 9

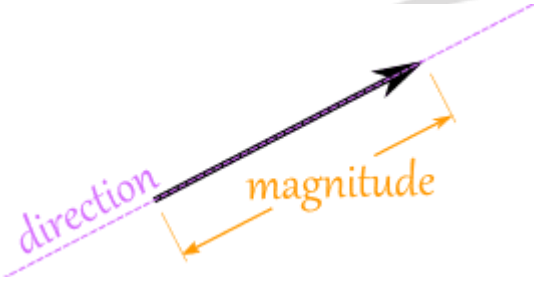
Complex Numbers in Non-Trivial Situations

Unit Standards	
7466	**Represent and operate on complex numbers in non-trivial situations**
Specific Outcomes	
SO1: Use complex numbers and systems to make sense of and solve non-trival problems **SO2: Carry out operations on complex numbers in non-trivial cases**	
Learning Outcomes	
• Understand, use and convert vectors, polar and rectangular forms	
Critical Cross-field Outcomes	
• Identifying • Collecting	• Communicating

VECTORS

This is a vector:

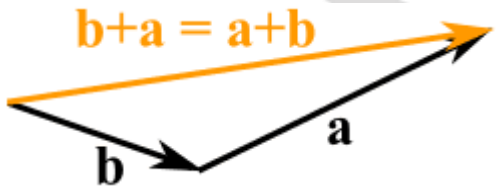
A vector has **magnitude** (how long it is) and **direction**:



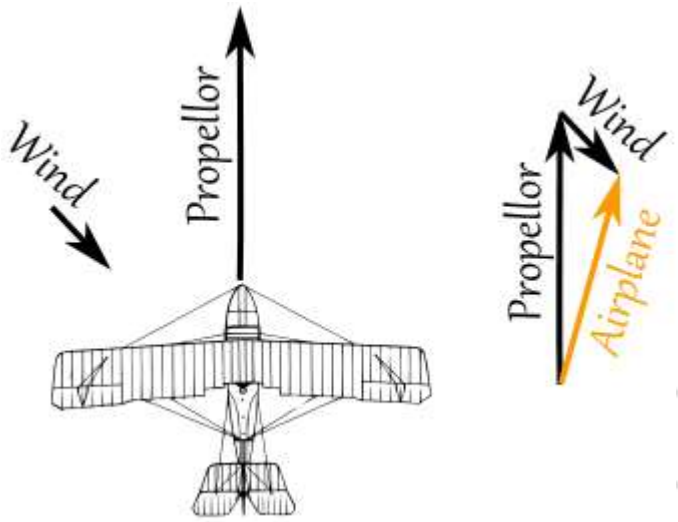
The length of the line shows its magnitude and the arrowhead points in the direction.

You can add two vectors by simply joining them head-to-tail:

And it doesn't matter which order you add them, you get the same result:

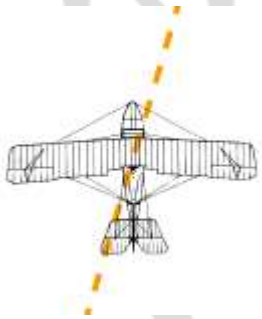


Example: A plane is flying along, pointing North, but there is a wind coming from the North-West.



The two vectors (the velocity caused by the propeller, and the velocity of the wind) result in a slightly slower ground speed heading a little East of North.

If you watched the plane from the ground it would seem to be slipping sideways a little.

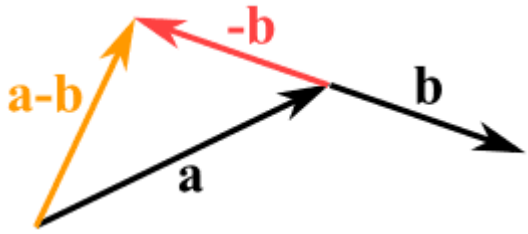


Have you ever seen that happen? Maybe you have seen birds struggling against a strong wind that seem to fly sideways. Vectors help explain that.

Subtracting

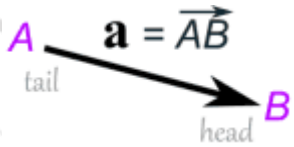
You can also subtract one vector from another:

- first you reverse the direction of the vector you want to subtract,
- then add them as usual:



Other Notation

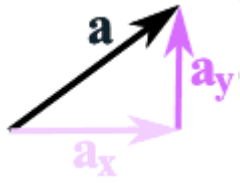
A vector can also be written as the letters of its head and tail with an arrow above, like this:



Calculations

Now ... how do we do the calculations?

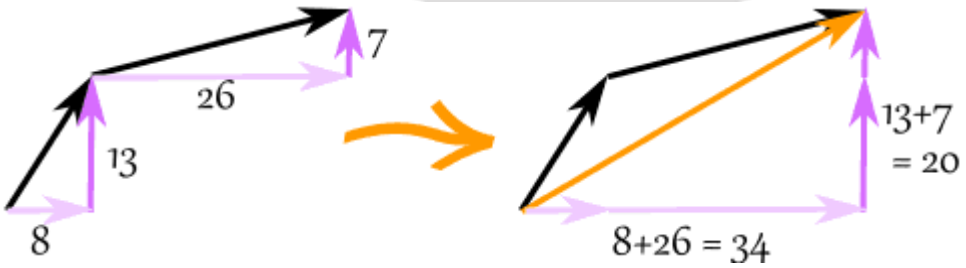
The most common way is to break up a vector into x and y pieces, like this:



The vector **a** is broken up into the two vectors $\mathbf{a}_x$ and $\mathbf{a}_y$

Adding Vectors

And here is how to add two vectors after breaking them into x and y parts:



The vector (8,13) and the vector (26,7) add up to the vector (34,20)

Example: add the vectors **a** = (8,13) and **b** = (26,7)

c = **a** + **b**

c = (8,13) + (26,7) = (8+26,13+7) = (34,20)

Subtracting Vectors

Remember: to subtract, first reverse the vector you want to subtract, then add.

Example: subtract **k** = (4,5) from **v** = (12,2)

a = **v** + -**k**

a = (12,2) + -(4,5) = (12,2) + (-4,-5) = (12-4,2-5) = (8,-3)

Magnitude of a Vector

The magnitude of a vector is shown by two vertical bars on either side of the vector:

|**a**|

OR it can be written with double vertical bars (so as not to confuse it with absolute value):

||**a**||

You can use Pythagoras' theorem to calculate it:

$|\mathbf{a}| = \sqrt{(x^2 + y^2)}$

A vector with magnitude 1 is called a Unit Vector.

Vector vs Scalar

When using vectors, we call an ordinary number a "scalar".

Scalar: just a number (like 7 or -0.32) ... definitely not a vector.

A vector is often written in **bold**,

so **c** is a vector, and it has magnitude and direction

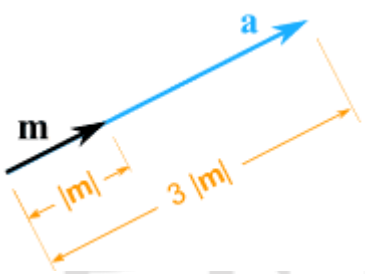
but c is just a value, like 3 or 12.4

Example: k**b** is actually the scalar k times the vector **b**.

Multiplying a Vector by a Scalar

When you multiply a vector by a scalar it is called "scaling" a vector, because you change how big or small the vector is.

Example: multiply the vector **m** = (7,3) by the scalar 3



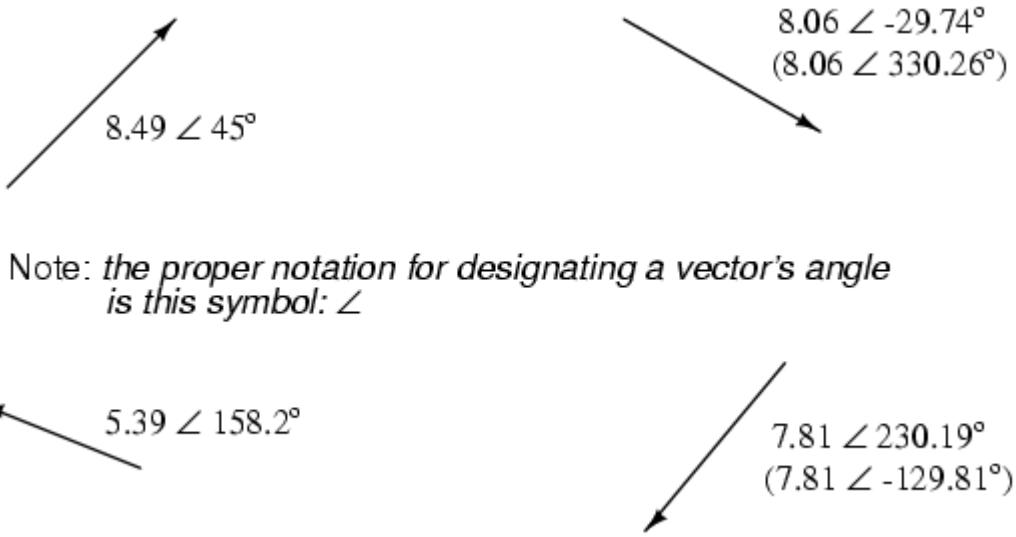
a = 3**m** = (3×7,3×3) = (21,9)

It still points in the same direction, but is 3 times longer

POLAR AND RECTANGULAR FORMS

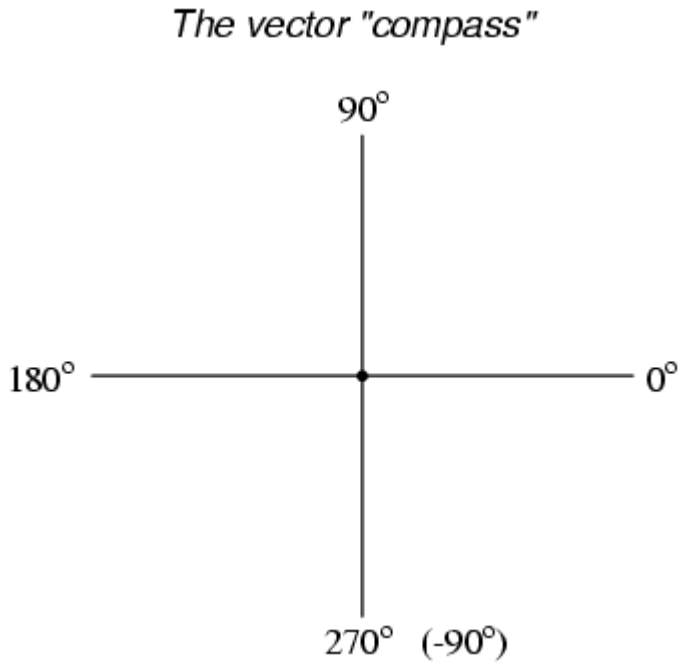
In order to work with these complex numbers without drawing vectors, we first need some kind of standard mathematical notation. There are two basic forms of complex number notation: *polar* and *rectangular*.

Polar form is where a complex number is denoted by the *length* (otherwise known as the *magnitude*, *absolute value*, or *modulus*) and the *angle* of its vector (usually denoted by an angle symbol that looks like this: ∠). To use the map analogy, polar notation for the vector from New York City to San Diego would be something like "2400 miles, southwest." Here are two examples of vectors and their polar notations: (Figure below)



Vectors with polar notations.

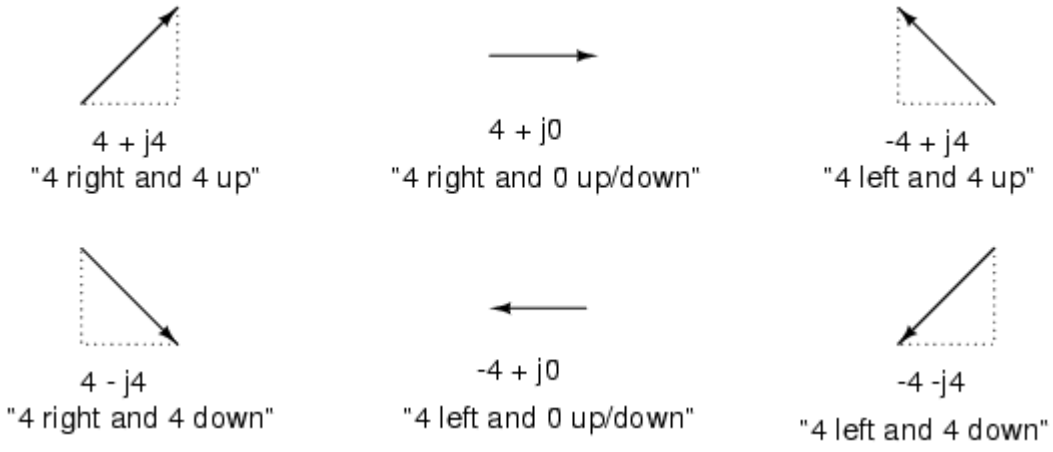
Standard orientation for vector angles in AC circuit calculations defines 0° as being to the right (horizontal), making 90° straight up, 180° to the left, and 270° straight down. Please note that vectors angled "down" can have angles represented in polar form as positive numbers in excess of 180, or negative numbers less than 180. For example, a vector angled ∠ 270° (straight down) can also be said to have an angle of -90°. (Figure below) The above vector on the right (7.81 ∠ 230.19°) can also be denoted as 7.81 ∠ -129.81°.



The vector compasses

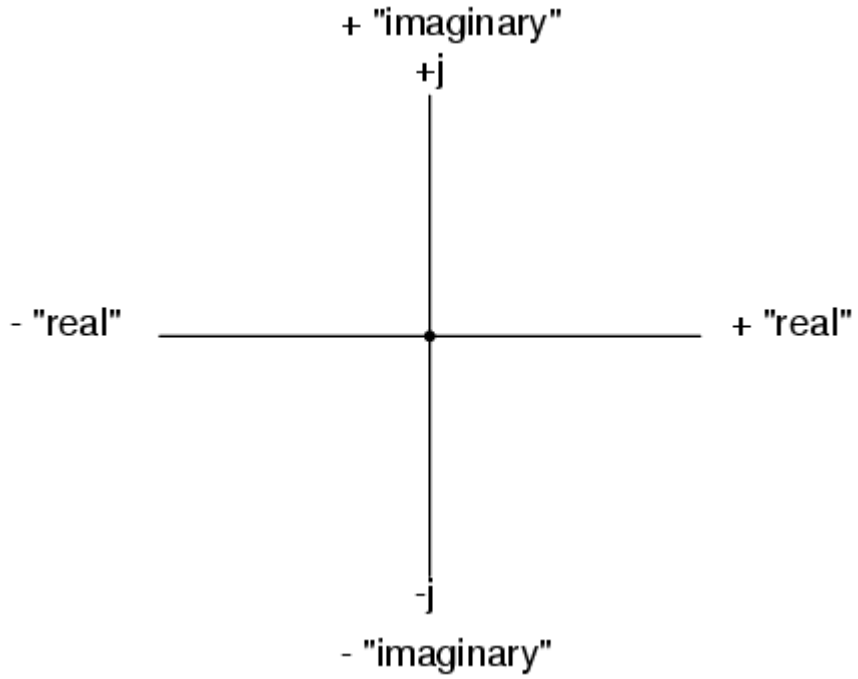
Rectangular form, on the other hand, is where a complex number is denoted by its respective horizontal and vertical components. In essence, the angled vector is taken to be the hypotenuse of a right triangle, described by the lengths of the adjacent and opposite sides. Rather than describing a vector's length and direction by denoting magnitude and angle, it is described in terms of "how far left/right" and "how far up/down."

These two-dimensional figures (horizontal and vertical) are symbolized by two numerical figures. In order to distinguish the horizontal and vertical dimensions from each other, the vertical is prefixed with a lower-case "i" (in pure mathematics) or "j" (in electronics). These lower-case letters do not represent a physical variable (such as instantaneous current, also symbolized by a lower-case letter "i"), but rather are mathematical *operators* used to distinguish the vector's vertical component from its horizontal component. As a complete complex number, the horizontal and vertical quantities are written as a sum: (Figure below)



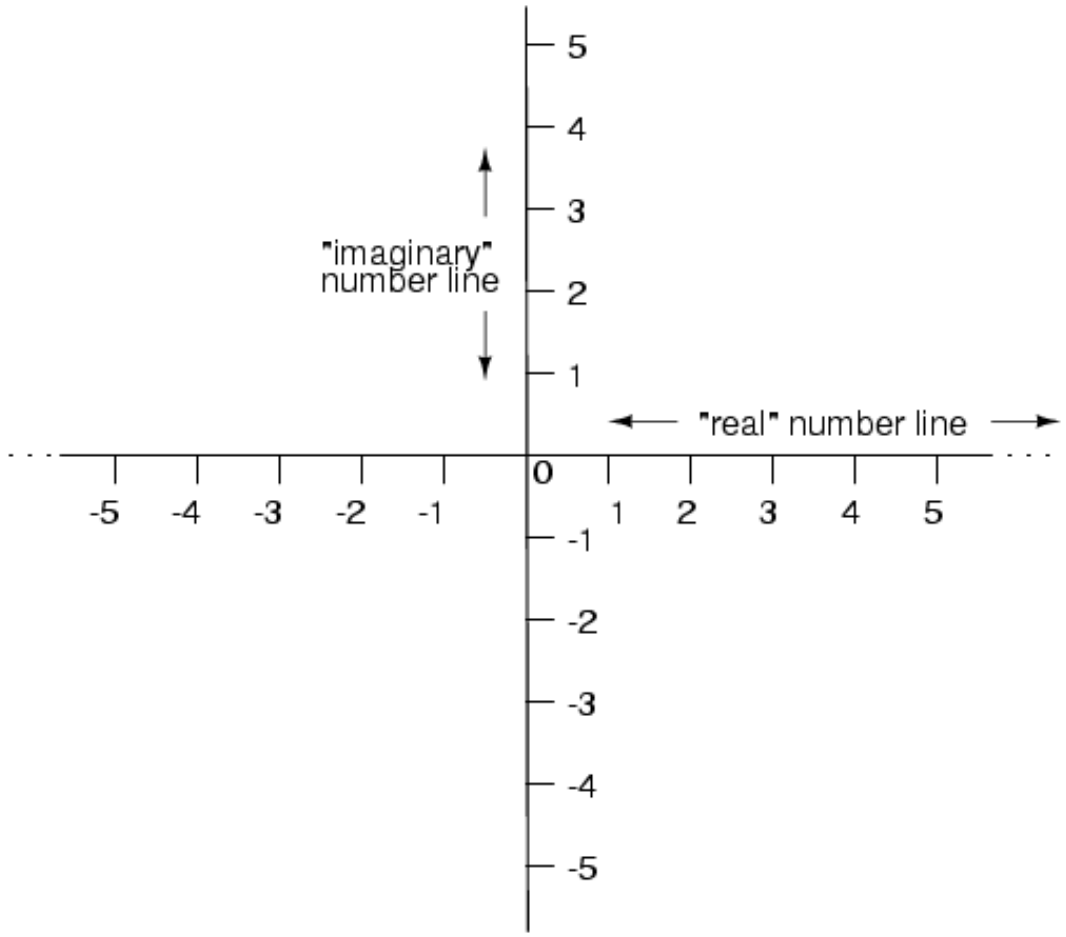
In "rectangular" form the vector's length and direction are denoted in terms of its horizontal and vertical span, the first number representing the horizontal ("real") and the second number (with the "j" prefix) representing the vertical ("imaginary") dimensions.

The horizontal component is referred to as the *real* component, since that dimension is compatible with normal, scalar ("real") numbers. The vertical component is referred to as the *imaginary* component, since that dimension lies in a different direction, totally alien to the scale of the real numbers. (Figure below)



Vector compass showing real and imaginary axes

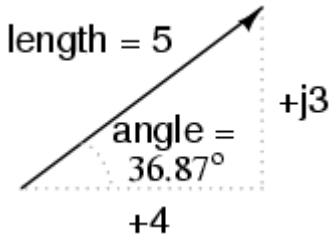
The "real" axis of the graph corresponds to the familiar number line we saw earlier: the one with both positive and negative values on it. The "imaginary" axis of the graph corresponds to another number line situated at 90° to the "real" one. Vectors being two-dimensional things, we must have a two-dimensional "map" upon which to express them, thus the two number lines perpendicular to each other: (Figure below)



Vector compass with real and imaginary ("j") number lines.

Either method of notation is valid for complex numbers. The primary reason for having two methods of notation is for ease of longhand calculation, rectangular form lending itself to addition and subtraction, and polar form lending itself to multiplication and division.

Conversion between the two notational forms involves simple trigonometry. To convert from polar to rectangular, find the real component by multiplying the polar magnitude by the cosine of the angle, and the imaginary component by multiplying the polar magnitude by the sine of the angle. This may be understood more readily by drawing the quantities as sides of a right triangle, the hypotenuse of the triangle representing the vector itself (its length and angle with respect to the horizontal constituting the polar form), the horizontal and vertical sides representing the "real" and "imaginary" rectangular components, respectively: (Figure below)



Magnitude vector in terms of real (4) and imaginary (j3) components.

$5 \angle 36.87°$ **(polar form)**

$(5)(\cos 36.87°) = 4$ (real component)

$(5)(\sin 36.87°) = 3$ (imaginary component)

$4 + j3$ **(rectangular form)**

To convert from rectangular to polar, find the polar magnitude through the use of the Pythagorean Theorem (the polar magnitude is the hypotenuse of a right triangle, and the real and imaginary components are the adjacent and opposite sides, respectively), and the angle by taking the arctangent of the imaginary component divided by the real component:

$4 + j3$ **(rectangular form)**

$c = \sqrt{a^2 + b^2}$ (pythagorean theorem)

$\text{polar magnitude} = \sqrt{4^2 + 3^2}$

$\text{polar magnitude} = 5$

$\text{polar angle} = \arctan \frac{3}{4}$

$\text{polar angle} = 36.87°$

$5 \angle 36.87°$ **(polar form)**

REVIEW:

- *Polar* notation denotes a complex number in terms of its vector's length and angular direction from the starting point. Example: fly 45 miles ∠ 203° (West by Southwest).
- *Rectangular* notation denotes a complex number in terms of its horizontal and vertical dimensions. Example: drive 41 miles West, then turn and drive 18 miles South.
- In rectangular notation, the first quantity is the "real" component (horizontal dimension of vector) and the second quantity is the "imaginary" component (vertical dimension of vector). The imaginary component is preceded by a lower-case "j," sometimes called the *j operator*.
- Both polar and rectangular forms of notation for a complex number can be related graphically in the form of a right triangle, with the hypotenuse representing the vector itself (polar form: hypotenuse length = magnitude; angle with respect to horizontal side = angle), the horizontal side representing the rectangular "real" component, and the vertical side representing the rectangular "imaginary" component.

Individual Formative Exercise 9A: Vectors

Learning Unit 10

Measurements and Calculations

Unit Standards	
12417	Measure, estimate & calculate physical quantities & explore, critique & prove geometrical relationships in 2 and 3-dimensional space in the life and workplace of adult with increasing responsibilities
7470	Work with a wide range of patterns and inverses of functions and solve related problems

Specific Outcomes

12417: SO1: Measure, estimate and calculate physical quantities

SO2: Explore, analyse and critique, describe and represent, interpret and justify geometric relationships

7470: SO1: Express and justify mathematical generalisations of situations.

SO2: Express mathematical functions and relationships.

SO3: Analyse and represent mathematical situations and structures using symbolic forms.

SO4: Use mathematical models to represent and deal with problems that arise in real and abstract contexts

Learning Outcomes

By the end of this learning unit you will demonstrate an understanding of:

- Measuring instruments and methods
- Pythagoras theorem
- Cartesian coordinate system
- Calculating volume and surface areas of geometrical shapes
- Scale drawings

Critical Cross-field Outcomes

Identify Collect	Communicate

INTRODUCTION

The purpose of this learning unit is to introduce you to ways of measuring, calculating and estimating physical quantities relevant to a warehouse. We will also explore, analyse, critique, describe and represent certain geometric relationships.

MEASUREMENT

Measurement is an important part of science, construction, art, design and a wide array of other professional fields. There are hundreds of measurement tools. Each measuring instrument serves a specific purpose for the person who is using it. There are a few measuring instruments that are more commonly seen than others.

Rulers, Yardsticks and Meter Sticks

Rulers are used to measure length, as are meter sticks and yardsticks. Rulers are used extensively in design labs and classrooms, while meter and yardsticks are more commonly used for construction purposes. A ruler measures in inches and is twelve inches long. A yardstick measures feet, inches and yards and is three feet long, while a meter stick measures meters, centimetres and millimetres and is one hundred centimetres long.

Beakers, Graduated Cylinders, and Cups

Beakers, graduated cylinders and measuring cups are used to measure the volume of a liquid. Measuring cups are most traditionally found in the kitchen to measure ingredients, while beakers and graduated cylinders are usually found in a science lab. While measuring cups use measurements like tablespoons, teaspoons and cups, beakers and graduated cylinders use the metric system and measure in millilitres and litres.

Scales and Balances

Scales and balances are used for yet another kind of measurement -- the measurement of an object's weight. Balances usually have two suspended baskets. On one side, a person places the object he or she wishes to measure. On the other side, weighted cubes are added until the two sides of the balance sit evenly. However, much weight has been placed on the measurement side is how much the object weighs. Scales operate in a similar manner but do not require weight to be added and simply make the calculation through internal software or sliding weights.

Standard measuring instruments used for scientific purposes

Standard measuring instruments used for scientific purposes to measure the physical properties of solids, gases or liquids come in a variety of sizes and dimensions for different science laboratory uses. Most science labs have a range of standard measuring instruments as well as some more specialized

measuring devices depending on the area of science being investigated.

Temperature

A thermascope or Galileo thermometer is a sealed glass tube containing a clear liquid and is different from the standard mercury-filled thermometer, both of which are used to measure temperature in a laboratory setting. Alcohol thermometers are used for measuring low temperatures and are known as spirit thermometers; they work in a similar way to mercury thermometers. Digital thermometers are also used in science labs and are the most technologically advanced temperature measuring instruments available.

Weight and Mass

There are various types of scale and balance instruments used for weighing in science. A triple beam balance scale allows you to weigh things with an accuracy of 0.1 of a grams and is ideal for most scientific purposes. Electronic postal scales can also be used for the same purpose, but traditionally the triple beam balance scale is the accepted instrument for science. The tubular spring scale is also a commonly used instrument for weighing objects in newton, pounds, grams and ounces.

Length and Thickness

A micrometer screw gauge is like a calliper, except it measures smaller lengths necessary in scientific experiments, and precise readings of one ten-thousandth of an inch can be accurately made. Micrometers come in a variety of designs and models. The micrometer was invented in the 17th century by an English astronomer and mathematician to measure the distance between stars; it was later adapted to measure smaller dimensions. A micrometer can measure the thickness of blocks, slot depth and shaft diameters.

Angles

A theodolite, also called an altimeter, is an instrument used for measuring angles from zenith -- both vertical and horizontal angles -- and is often used in mining. The instrument has a graduated telescopic eyepiece and measures the height or altitude above a reference point. Theodolites are used by surveyors to plot out a specific area using the angles as reference points.

Viscosity

A viscometer is an instrument used to measure the viscosity of liquids; the most common variety used is an Ostwald viscometer, also known as a u-tube viscometer. Rotational viscometers, another type of viscometer, are used for laboratory viscometry in food science laboratories. They use torque to rotate a spindle at a constant speed while immersed in a liquid, thereby measuring the viscosity of liquids.

Activity: Think of the application of these measuring instruments on the FARM. Name a few possible uses:

_____

_____

_____

Pythagoras theorem

Years ago, a man named Pythagoras found an amazing fact about triangles:

If the triangle had a right angle (90°) ...

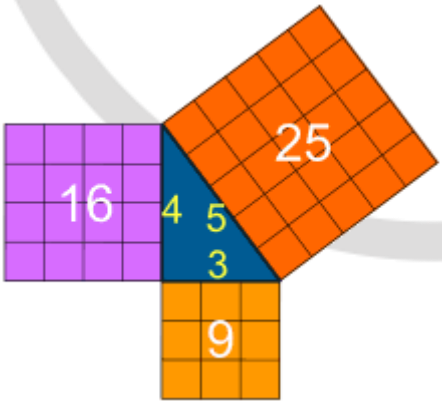
... and you made a square on each of the three sides, then ...

*... the biggest square had the **exact same area** as the other two squares put together!*

Pythagoras' theorem is mainly used to calculate height and distance for example

Definition

The longest side of the triangle is called the "hypotenuse", so the formal definition is:
In a right-angled triangle:
the square of the hypotenuse is equal to
the sum of the squares of the other two sides.
Let's see if it really works using an example.



Let's check if the areas **are** the same:

$3^2 + 4^2 = 5^2$

Calculating this becomes:

$9 + 16 = 25$

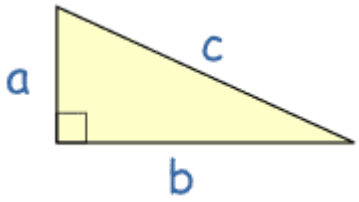
It works ... like Magic!

Why Is This Useful?

If we know the lengths of **two sides** of a right-angled triangle, we can find the length of the **third side**. (But remember it only works on right angled triangles!)

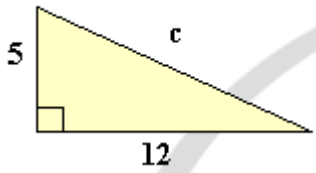
How Do I Use it?

Write it down as an equation:



$a^2 + b^2 = c^2$

Now you can use algebra to find any missing value, as in the following examples:



$a^2 + b^2 = c^2$

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

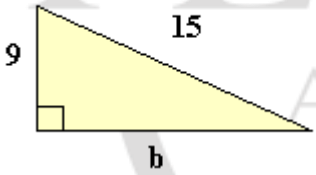
$169 = c^2$

$c^2 = 169$

$c = \sqrt{169}$

c = 13

You can also read about Squares and Square Roots to find out why $\sqrt{169} = 13$



$a^2 + b^2 = c^2$

$9^2 + b^2 = 15^2$

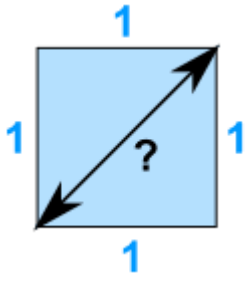
$81 + b^2 = 225$

Take 81 from both sides:

$b^2 = 144$

$b = \sqrt{144}$

b = 12



$a^2 + b^2 = c^2$

$1^2 + 1^2 = c^2$

$1 + 1 = c^2$

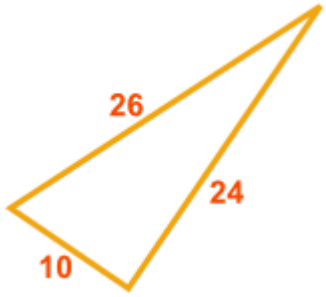
$2 = c^2$

$c^2 = 2$

$c = \sqrt{2} = 1.4142...$

It works the other way around, too: when the three sides of a triangle make $a^2 + b^2 = c^2$, then the triangle is right angled.

Does this triangle have a Right Angle?



Does $a^2 + b^2 = c^2$?

$a^2 + b^2 = 10^2 + 24^2 = 100 + 576 =$ **676**

$c^2 = 26^2 =$ **676**

They are equal, so ...

Yes, it does have a Right Angle!

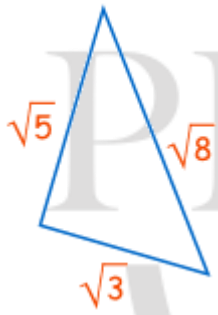
Does an 8, 15, 16 triangle have a Right Angle?

Does $8^2 + 15^2 = 16^2$?

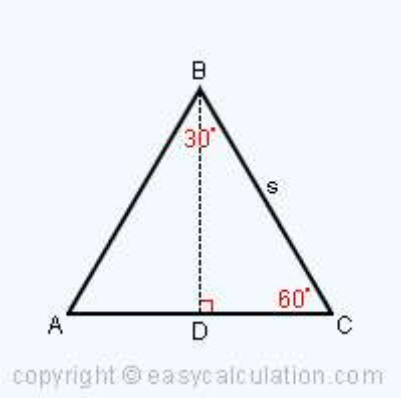
- $8^2 + 15^2 = 64 + 225 =$ **289**,
- but $16^2 =$ **256**

So, NO, it does not have a Right Angle

Example: Does this triangle have a Right Angle?



In the below example, let the length of the sides be 4 cm. Draw the altitude from B forming BD. The altitudes drawn in an equilateral and isosceles triangle touch the midpoint of the opposite side dividing the side into two equal halves and form two right-angled triangles.



Equivalent triangle

Let us take BDC to find the height (BD) with DC being 2 cm (half of AC).

$$BC^2 = BD^2 + DC^2$$

$$4^2 = BD^2 + 2^2$$

$$BD^2 = 16 - 4$$

$$BD^2 = 12$$

$$BD = 3.46$$

Hence the height of the above equilateral is 3.46 cm.

Activity: Name a few uses of this formula on the FARM:

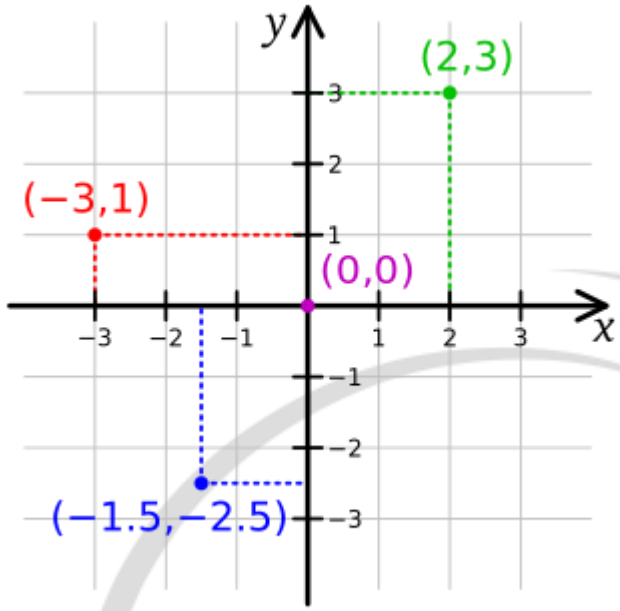
_____

_____

_____

A Cartesian coordinate system

A **Cartesian coordinate system** specifies each point uniquely in a plane by a pair of numerical **coordinates**, which are the signed distances from the point to two fixed perpendicular directed lines, measured in the same unit of length. Each reference line is called a *coordinate axis* or just *axis* of the system, and the point where they meet is its *origin*, usually at ordered pair (0,0). The coordinates can also be defined as the positions of the perpendicular projections of the point onto the two axes, expressed as signed distances from the origin.



Activity: Name a few uses of this system on the FARM:

_____

_____

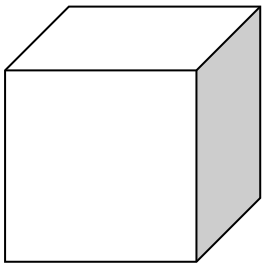
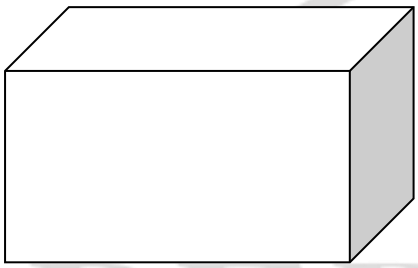
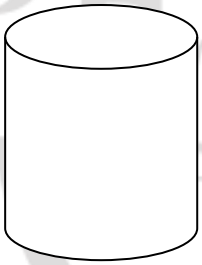
_____

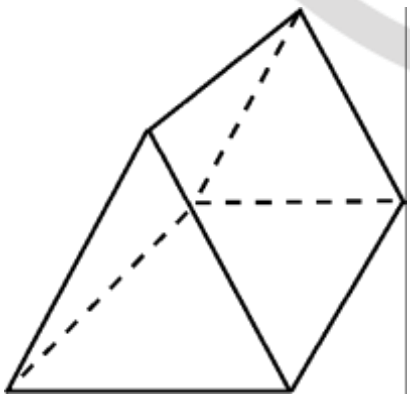
Calculating Volumes and Surface Areas

When we calculate volume, it is the amount of liquid or air that a 3-dimensional shape can hold.

A surface area on the other hand, is the exterior surface of the shape that you could paint.

Here are the formulae to use:

Shape	Volume	Surface area
Cube: 	V = Base of square area x height $= (x \times x) \times x$ $= x^3$	SA = $6 \times (x \times x)$ $= 6x^2$
Rectangular Prism 	V = length x breadth x height	SA =2length breadth + 2breadth height + 2 length height =2(lb + bh + lh)
Cylinder 	V =Area of circular base x height $= \pi r^2 \times H$	SA $= 2\pi r^2 + curve\ rectangle$ $= 2\pi r^2 + 2\pi rh$ $= 2\pi r\ (r + h)$

Triangular prism	V = area of rectangular base x height	SA = 2 triangles + 3 rectangles
	$= \left(\frac{1}{2}\ b \times h\right) \times H$	$= 2\left(\frac{1}{2}b \times h\right) + (x + y + z)H$ $= b \times h + (perimeter \times H)$

Activity: Name a few uses of this system on your FARM:

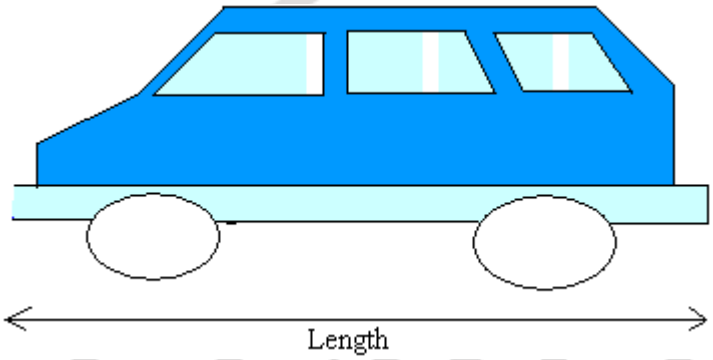
_____

_____

_____

Scale drawings

Since it is not always possible to draw on paper the actual size of real-life objects such as the real size of a car, an airplane, we need scale drawings to represent the size like the one you see below of a van.



In real-life, the length of this van may measure 240 inches. However, the length of a copy or print paper that you could use to draw this van is a little bit less than 12 inches

Since 240/12 = 20, you will need about 20 sheets of copy paper to draw the length of the actual size of the van

In order to use just one sheet, you could then use 1 inch on your drawing to represent 20 inches on the real-life object

You can write this situation as 1:20 or 1/20 or 1 to 20

Notice that the first number always refers to the length of the drawing on paper and the second number refers to the length of real-life object

Example 1:

Suppose a problem tells you that the length of a vehicle is drawn to scale. The scale of the drawing is 1:20

If the length of the drawing of the vehicle on paper is 12 inches, how long is the vehicle in real life?

Set up a proportion that will look like this:

$$\frac{\text{Length of drawing}}{\text{Real length}} = \frac{1}{20}$$

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction

We get:

Length of drawing × 20 = Real length × 1

Since length of drawing = 12, we get:

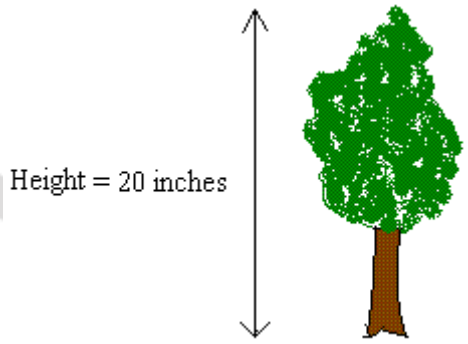
12 × 20 = Real length × 1

240 inches = Real length

Example 2:

The scale drawing of this tree is 1:500

If the height of the tree on paper is 20 inches, what is the height of the tree in real life?



Set up a proportion like this:

$$\frac{\text{Height of drawing}}{\text{Real height}} = \frac{1}{500}$$

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction

We get:

Height of drawing × 500 = Real height × 1

Since height of drawing = 20, we get:

20 × 500 = Real length × 1

10000 inches = Real height

Activity: Name a few uses of these drawings on the FARM:

_____

_____

_____

Individual Formative Exercise 10A

Arithmetic Sequences

An Arithmetic Sequence is made by **adding** some value each time.

Example:

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number.
The pattern is continued by **adding 3** to the last number each time.

Example:

3, 8, 13, 18, 23, 28, 33, 38, ...

This sequence has a difference of 5 between each number.
The pattern is continued by **adding 5** to the last number each time.

The value added each time is called the "common difference"

What is the common difference in this example?

19, 27, 35, 43, ...

Answer: The common difference is **8**

The common difference could also be negative, like this:

25, 23, 21, 19, 17, 15, ...

This common difference is **-2**

The pattern is continued by **subtracting 2** each time.

Geometric Sequences

A Geometric Sequence is made by **multiplying** by some value each time.

Example:

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a factor of 2 between each number.
The pattern is continued by **multiplying by 2** each time.

Example:

3, 9, 27, 81, 243, 729, 2187, ...

This sequence has a factor of 3 between each number.
The pattern is continued by **multiplying by 3** each time.

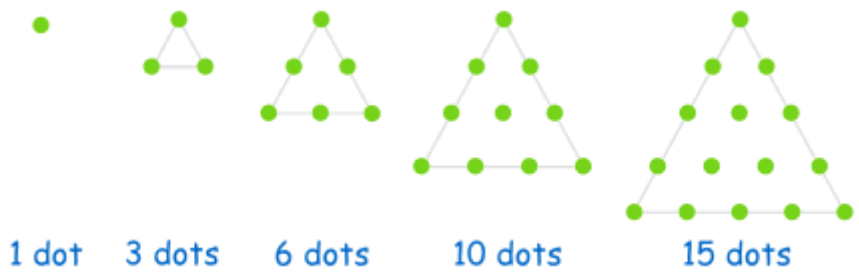
Special Sequences

Triangular Numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, ...

This Triangular Number Sequence is generated from a pattern of dots which form a triangle.

By adding another row of dots and counting all the dots we can find the next number of the sequence:



Square Numbers

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, ...

They are the squares of numbers:

0 (=0×0)
1 (=1×1)
4 (=2×2)
9 (=3×3)
16 (=4×4)
etc...

Cube Numbers

0, 1, 8, 27, 64, 125, 216, 343, 512, 729, ...

They are the cubes of numbers:

0 (=0×0×0)
1 (=1×1×1)
8 (=2×2×2)
27 (=3×3×3)
64 (=4×4×4)
etc...

Fibonacci Numbers

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The Fibonacci Sequence is found by adding the two numbers before it together.
The 2 is found by adding the two numbers before it (1+1)
The 21 is found by adding the two numbers before it (8+13)
The next number in the sequence above would be 55 (21+34)

Can you figure out the **next** few numbers?

Other Sequences

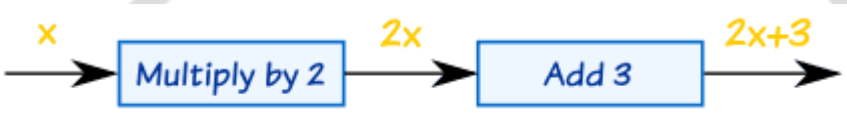
There are lots more! You might even think of your own ...

Inverse Functions

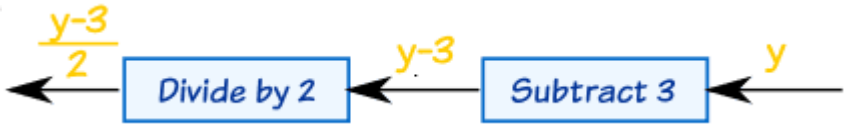
An inverse function goes in the opposite direction!

Let us start with an example:

Here we have the function **f(x) = 2x+3**, written as a flow diagram:



The Inverse Function just goes the other way:



So the inverse of: 2x+3 is: (y-3)/2

The inverse is usually shown by putting a little "-1" after the function name, like this:

$f^{-1}(y)$

*We say "**f inverse** of y"*

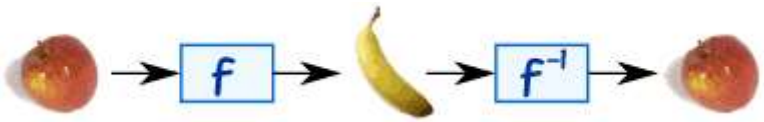
So, the inverse of f(x) = 2x+3 is written:

$f^{-1}(y) = (y-3)/2$

(I also used **y** instead of **x** to show that we are using a different value.)

Back to Where We Started

The cool thing about the inverse is that it should give you back the original value:



If the function f turns the apple into a banana,

Then the **inverse** function f^{-1} turns the banana back to the apple

Example:

Using the formulas from above, we can start with x=4:

$$f(4) = 2\times4+3 = 11$$

We can then use the inverse on the 11:

$$f^{-1}(11) = (11-3)/2 = 4$$

And we magically get **4** back again!

We can write that in one line:

$$f^{-1}(f(4)) = 4$$

"f inverse of f of 4 equals 4"

So, applying a function f and then its inverse f^{-1} gives us the original value back again:

$$f^{-1}(f(x)) = x$$

We could also have put the functions in the other order and it still works:

$$f(f^{-1}(x)) = x$$

Example:

Start with:

$$f^{-1}(11) = (11-3)/2 = 4$$

And then:

$$f(4) = 2\times4+3 = 11$$

So, we can say:

$$f(f^{-1}(11)) = 11$$

"f of f inverse of 11 equals 11"

Solve Using Algebra

You can work out the inverse using Algebra. **Put "y" for "f(x)" and solve for x:**

The function:	**f(x) = 2x+3**
Put "y" for "f(x)":	**y = 2x+3**
Subtract 3 from both sides:	**y-3 = 2x**
Divide both sides by 2:	**(y-3)/2 = x**
Swap sides:	**x = (y-3)/2**

Solution (put "$f^{-1}(y)$" for "x") : **$f^{-1}(y) = (y-3)/2$**

This method works well for more difficult inverses.

Fahrenheit to Celsius

A useful example is converting between Fahrenheit and Celsius:

To convert Fahrenheit to Celsius: **$f(F) = (F - 32) \times {}^{5}/_{9}$**

The **Inverse Function** (Celsius back to Fahrenheit) is: **$f^{-1}(C) = (C \times {}^{9}/_{5}) + 32$**

For You: see if you can do the steps to create that inverse!

Inverses of Common Functions

It has been easy so far, because we know the inverse of Multiply is Divide, and the inverse of Add is Subtract, but what about other functions?

Here is a list to help you:

Inverses			Careful!
+	<=>	−	
×	<=>	÷	Don't divide by zero
$\frac{1}{x}$	<=>	$\frac{1}{y}$	x and y not zero
x^2	<=>	$\sqrt{y}$	x and y ≥ 0

x^n	<=>	$\sqrt[n]{y}$ **or** $y^{\frac{1}{n}}$	n not zero *(different rules when n is odd, even, negative or positive)*
e^x	<=>	$ln(y)$	y > 0
a^x	<=>	$log_a(y)$	y and a > 0
$sin(x)$	<=>	$sin^{-1}(y)$	-π/2 to +π/2
$cos(x)$	<=>	$cos^{-1}(y)$	0 to π
$tan(x)$	<=>	$tan^{-1}(y)$	-π/2 to +π/2

(Note: you can read more about Inverse Sine, Cosine and Tangent.)

Careful!

Did you see the "Careful!" column above? That is because some inverses work **only with certain values**.

Example: Square and Square Root

If you square a **negative** number, and then do the inverse this happens:

Square: $(-2)^2 = 4$

Inverse (Square Root): $\sqrt{(4)} = 2$

But we didn't get the original value back! We got **2** instead of **-2**. Our fault for not being careful!

So, the square function (as it stands) **does not have an inverse**

But we can fix that!

Restrict the Domain (the values that can go into a function).

Example: (continued)

Just make sure you don't use negative numbers.

In other words, restrict it to **$x \geq 0$** and then you can have an inverse.

So we have this situation:

- x^2 does **not** have an inverse
- but $\{x^2 \mid x \geq 0\}$ (which says "x squared such that x is greater than or equal to zero" using set-builder notation) **does** have an inverse.

Why Would There Be No Inverse?

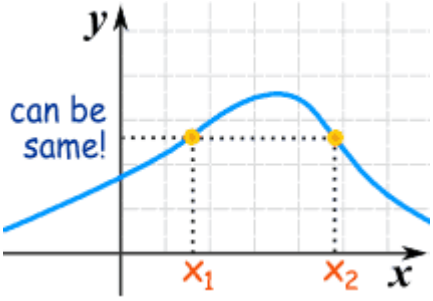
Let us see graphically what is going on here:

To be able to have an inverse you need **unique values**.

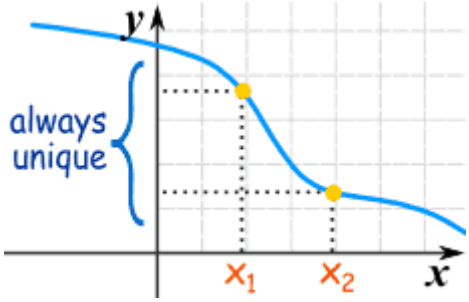
Just think ... if there are two or more **x-values** for one **y-value**, how do you know which one to choose when going back?

 No Inverse

 Inverse is Possible



When a y-value has more than one x-value, how do you know which x-value to go back to?



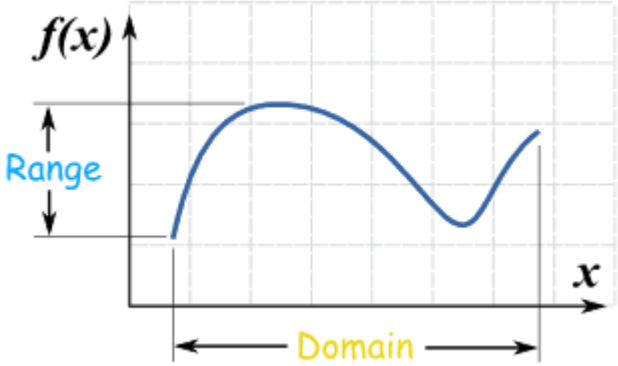
When there is a unique y-value for every x-value you can always "go back" from y to x.

So, we have this idea of "a unique y-value for every x-value", and it actually has a name. It is called "**Injective**" or "**One-to-one**":

If a function is "One-to-one" (Injective) it has an inverse.

Domain and Range

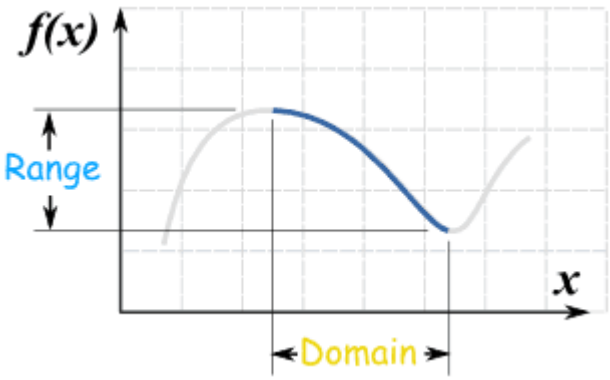
So what is all this talk about "**Restricting the Domain**"?



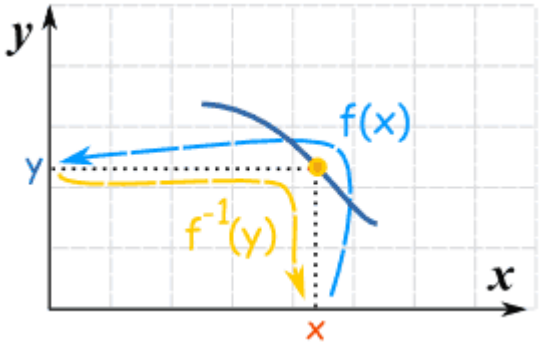
In its simplest form the **domain** is all the values that go into a function (and the **range** is all the values that come out).

As it stands the function above does **not** have an inverse.

But you could restrict the domain so there is a **unique y for every x** ...



... and **now** you can have an inverse:



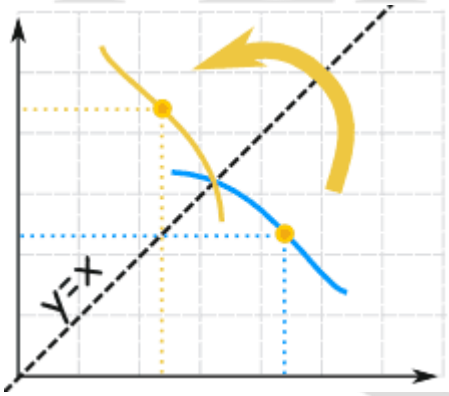
Note also:

The function **f(x)** goes from the domain to the range,

The inverse function $\mathbf{f^{-1}(y)}$ goes from the range back to the domain.

Or...

You could plot them both in terms of **x** ... so it is now $\mathbf{f^{-1}(x)}$, not $\mathbf{f^{-1}(y)}$.



f(x) and $\mathbf{f^{-1}(x)}$ are like mirror images (flipped about the diagonal).

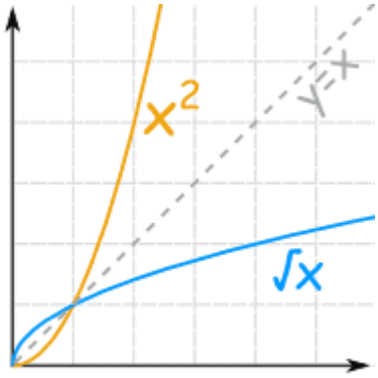
In other words:

The graph of **f(x)** and $\mathbf{f^{-1}(x)}$ are symmetric across the line **y=x**

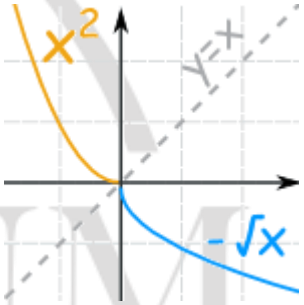
Example: Square and Square Root (continued)

First, we restrict the Domain to $\mathbf{x \geq 0}$:

- $\mathbf{\{x^2 \mid x \geq 0\}}$ *"x squared such that x is greater than or equal to zero"*
- $\mathbf{\{\sqrt{x} \mid x \geq 0\}}$ *"square root of x such that x is greater than or equal to zero"*



And you can see they are "mirror images"
of each other about the diagonal y=x.



Note: we could have restricted the domain to **x ≤ 0** and the inverse would then be $\mathbf{f^{-1}(x) = -\sqrt{x}}$:

$\mathbf{\{x^2 \mid x \leq 0\}}$

$\mathbf{\{-\sqrt{x} \mid x \geq 0\}}$

Which are inverses, too.

Not Always Solvable!

It is sometimes not possible to find an Inverse of a Function.

Example: f(x) = x/2 + sin(x)

We cannot work out the inverse of this, because we cannot solve for "x":

$$y = x/2 + \sin(x)$$

$$y \ldots ? = x$$

Notes on Notation

Even though we write $f^{-1}(x)$, the "-1" is **not** an exponent (or power):

$f^{-1}(x)$	***...is different to...***	$f(x)^{-1}$
Inverse of the function **f**		$\mathbf{f(x)^{-1} = 1/f(x)}$ (the Reciprocal)

Summary

- The inverse of f(x) is $f^{-1}(y)$
- You can find an inverse by reversing the "flow diagram"
- Or you can find an inverse by using Algebra:
  - Put "y" for "f(x)", and
  - Solve for x
- You may need to **restrict the domain** for the function to have an inverse

Individual Formative Exercise 10B

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